

# Appendix 4B:Circuit Non-ideal Effect Analysis

## Outlines

1. Design Example
2. Opamp DC gain and gain nonlinearity
3. Opamp bandwidth and slew rate
4. Capacitance nonlinearity
5. Coefficient mismatch
6. Noise Budget

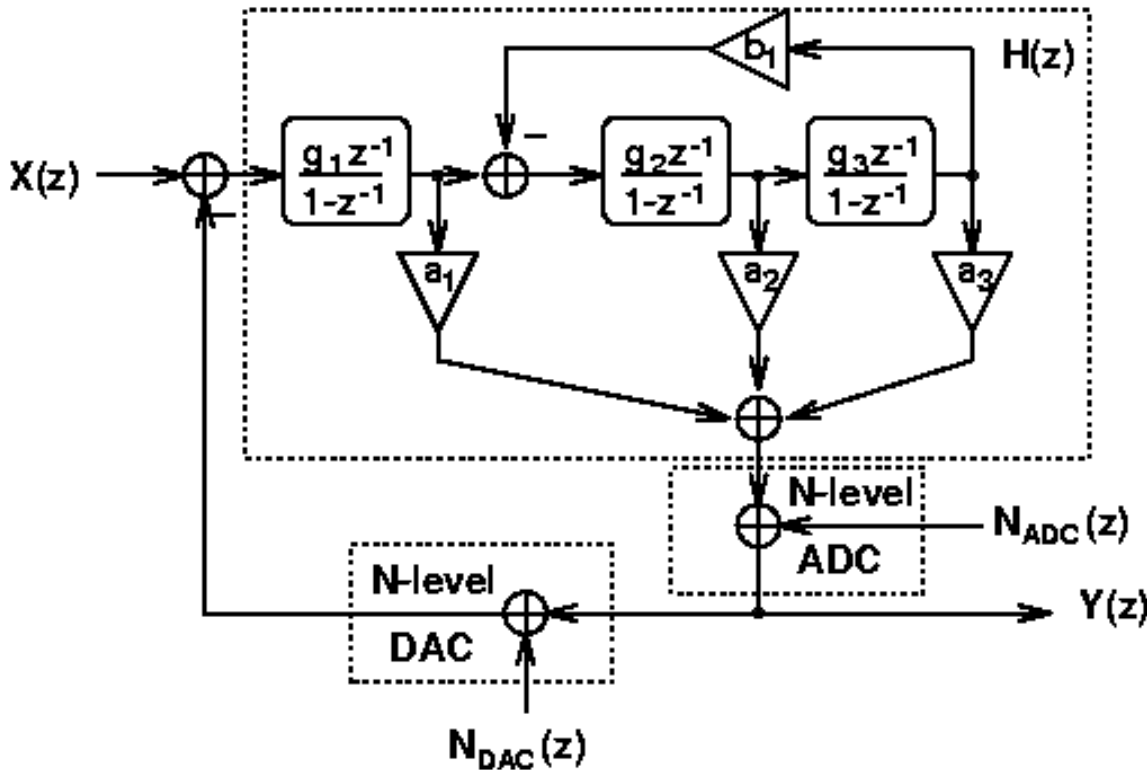
## Design Example

- A third-order 3-bit sigma-delta modulator
  - OSR=32
  - Bandwidth=20kHz
  - $F_s=1.28\text{MHz}$
  - PSNR=99dB (ideally)
  - DAC feedback reference(full scale)=  $\pm 1$
  - Maximum stable input (without clipping)= $\pm 0.75$
  - Maximum integrator output swing=  $\pm 1$
  - Integrator saturation level=  $\pm 1.5$

# Third-Order 3-Bit Sigma-Delta Modulator

- Block diagram

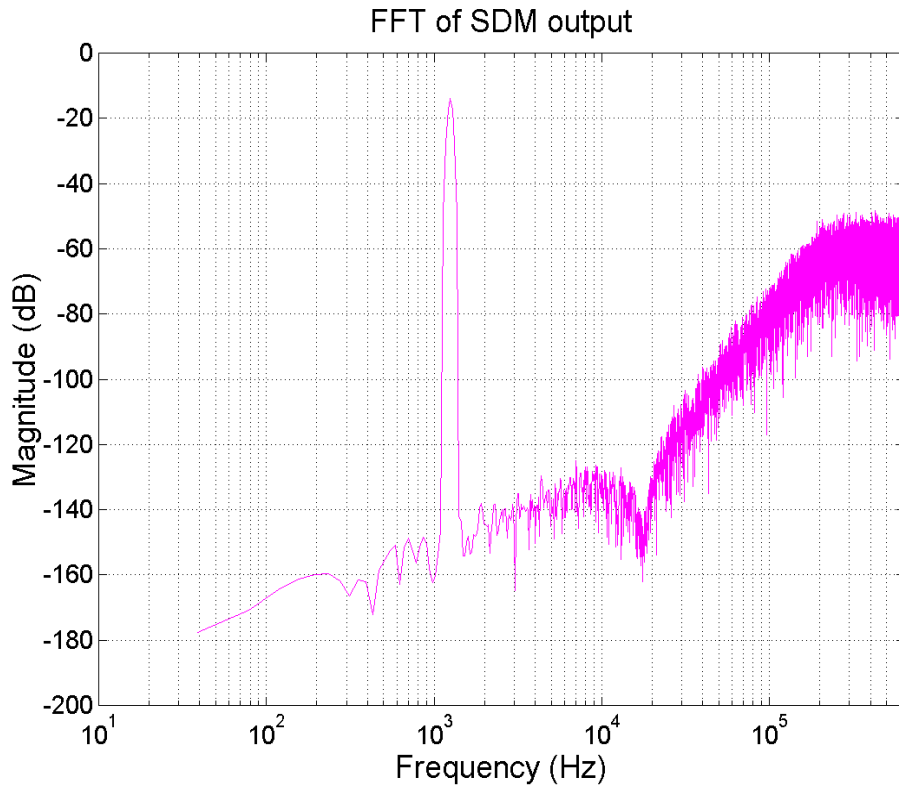
- Coefficients



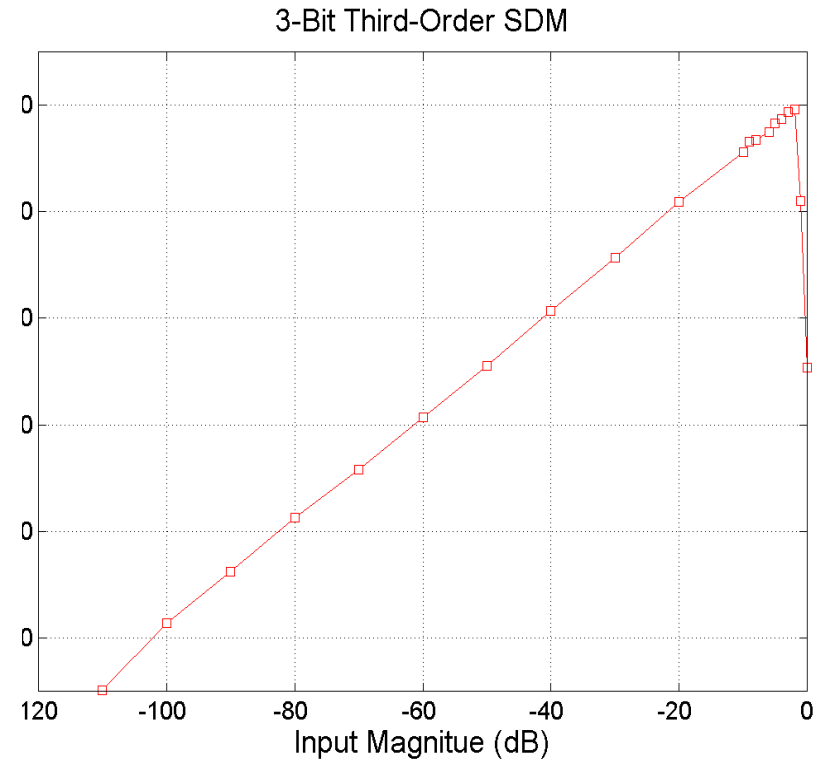
$g_1=2.9302$   
 $g_2=1.1850$   
 $g_3=0.2017$   
 $a_1=0.6510$   
 $a_2=0.4212$   
 $a_3=0.6025$   
 $b_1=0.0310$

# SNR Performance

- FFT plot

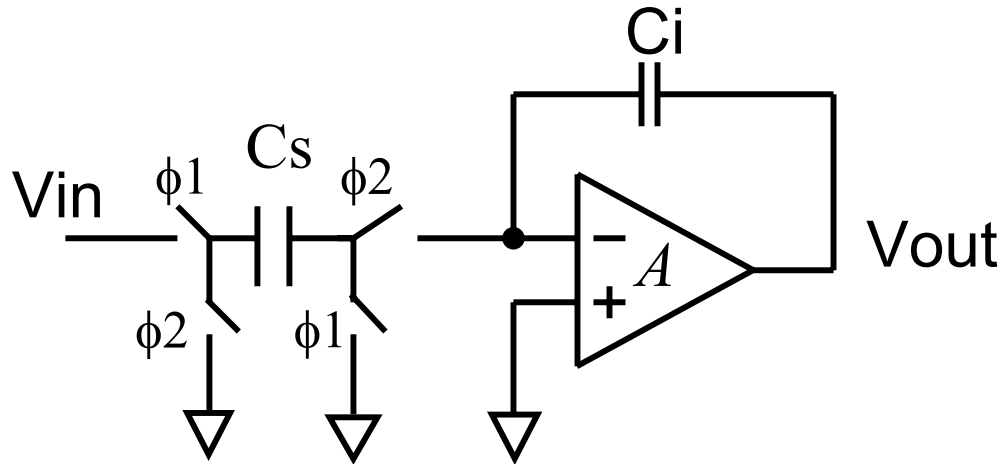


- SNR plot



# Opamp DC gain

- SC integrator



$$H(z) = \frac{C_i}{C_s} \frac{z^{-1}}{1 - z^{-1}} = g \frac{z^{-1}}{1 - z^{-1}} \quad \text{Ideal}$$

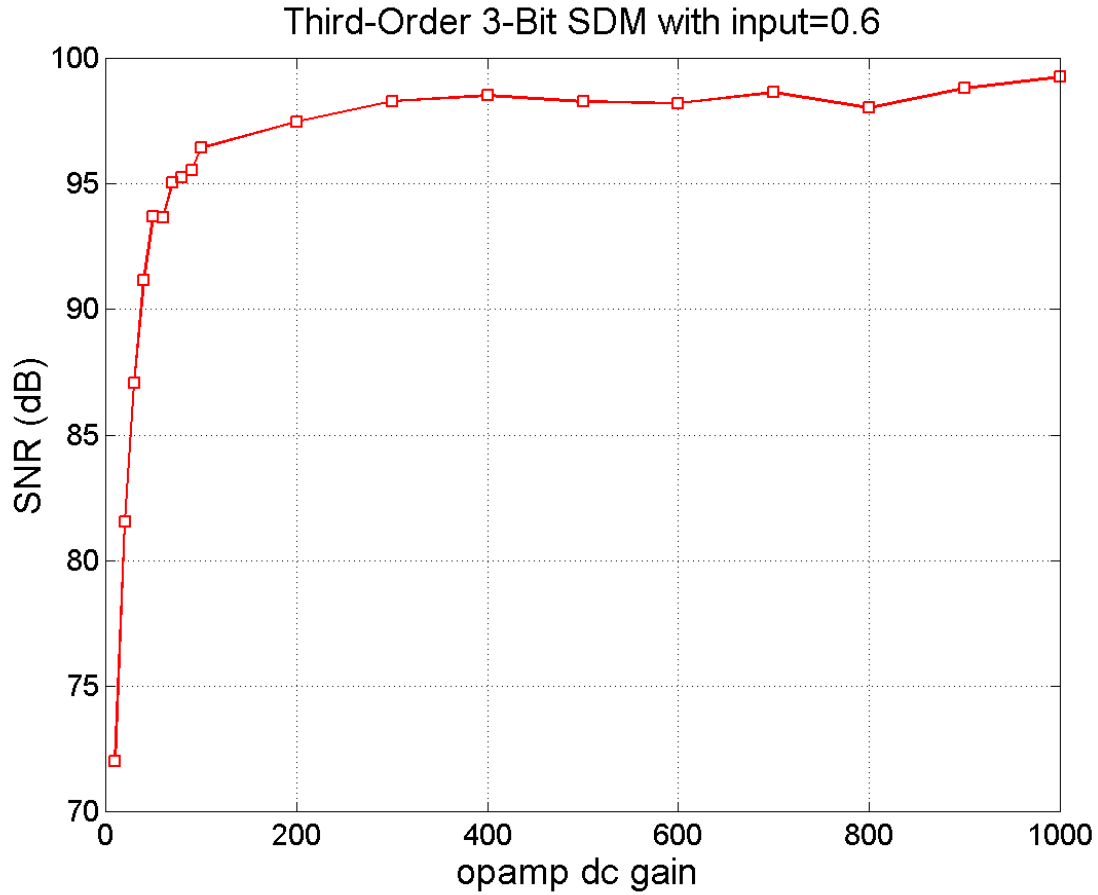
$$H(z) = \frac{\alpha z^{-1}}{1 - \beta z^{-1}} \quad \text{Nonideal}$$

– where

$$\alpha = \frac{g}{1 + \frac{1}{A}(1 + g)} \quad \beta = \frac{1 + \frac{1}{A}}{1 + \frac{1}{A}(1 + g)}$$

# Simulation

- Opamp gain > 400

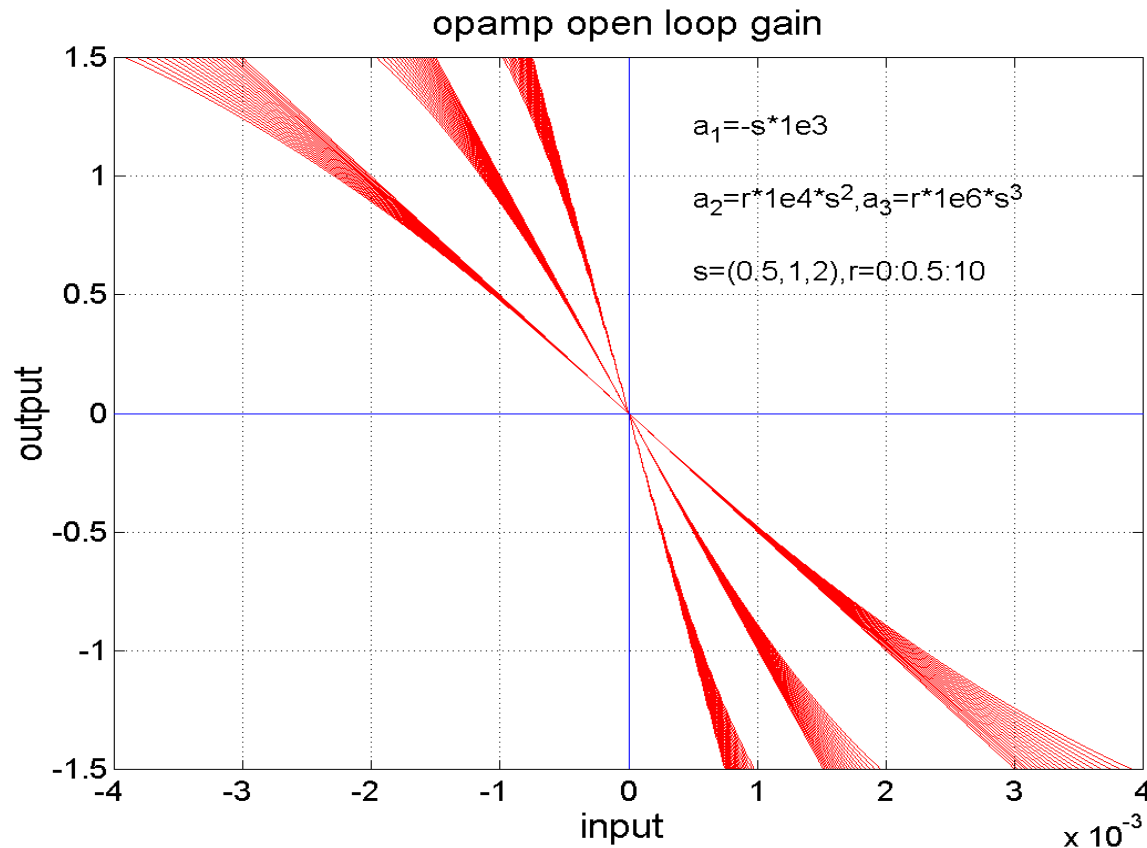


# Opamp Gain Nonlinearity

- Assume opamp open loop output/input

$$v_o = a_1 v_i + a_2 v_i^2 + a_3 v_i^3$$

- Open loop gain simulation



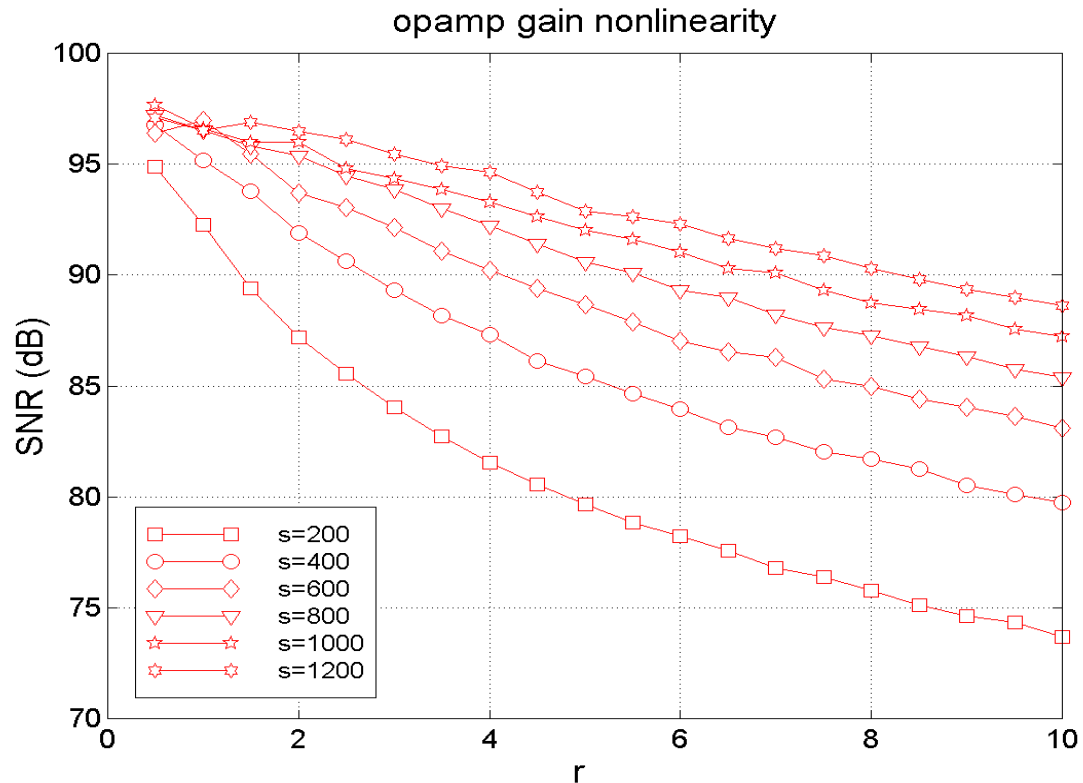
# Opamp Gain Nonlinearity(cont.)

- Integrator output

$$v_o(n) - v_o(n-1) \cong g \left\{ v_i(n-1) \left[ 1 + \frac{1}{a_1} \right] - \frac{a_2}{a_1^3} [v_i(n-1)[v_o(n) + v_o(n-1)] - v_o^2(n)] \right.$$

$$\left. - \frac{a_3}{a_1^4} [v_i(n-1)[v_o^2(n) + v_o(n)v_o(n-1) + v_o^2(n-1)] - v_o^3(n) \right\}$$

- Simulation





# Opamp Gain Nonlinearity(cont.)

- Harmonic distortion

$$HD_2 = \frac{a_2}{a_1^3} v_o \sqrt{1 + \left(\frac{v_o}{2v_i}\right)^2}$$

$$HD_3 = \frac{a_3}{a_1^4} v_o^2 \sqrt{9 + \left(\frac{v_o}{v_i}\right)^2}$$

- Higher DC gain lowers the harmonic distortions due to opamp gain nonlinearity

# Opamp Bandwidth and Slew Rate

- First-order model of the settling characteristics of a SC integrator

$$v_o(n) = v_o(n-1) + g(v_i(n-1))$$

$$g(x) = x(1 - e^{-Ts/\tau}) \quad |x| \leq \tau S$$

$$= x - \text{sgn}(x)\tau S e^{(\frac{|x|}{\tau S} - \frac{T_s}{\tau} - 1)}$$

$$\tau S \leq |x| \leq (\tau + Ts)S$$

$$= \text{sgn}(x)STs \quad (\tau + Ts)S \leq |x|$$

$$S = nS_n, \quad S_n = g1 \cdot \Delta / Ts = 2.134 * 10^6 \quad \tau \cong \frac{Cc(1 + g1)}{g_m} = \frac{1 + g1}{\omega_t}$$

t : integrator time constant

Ts: sampling duration = 390.625nS

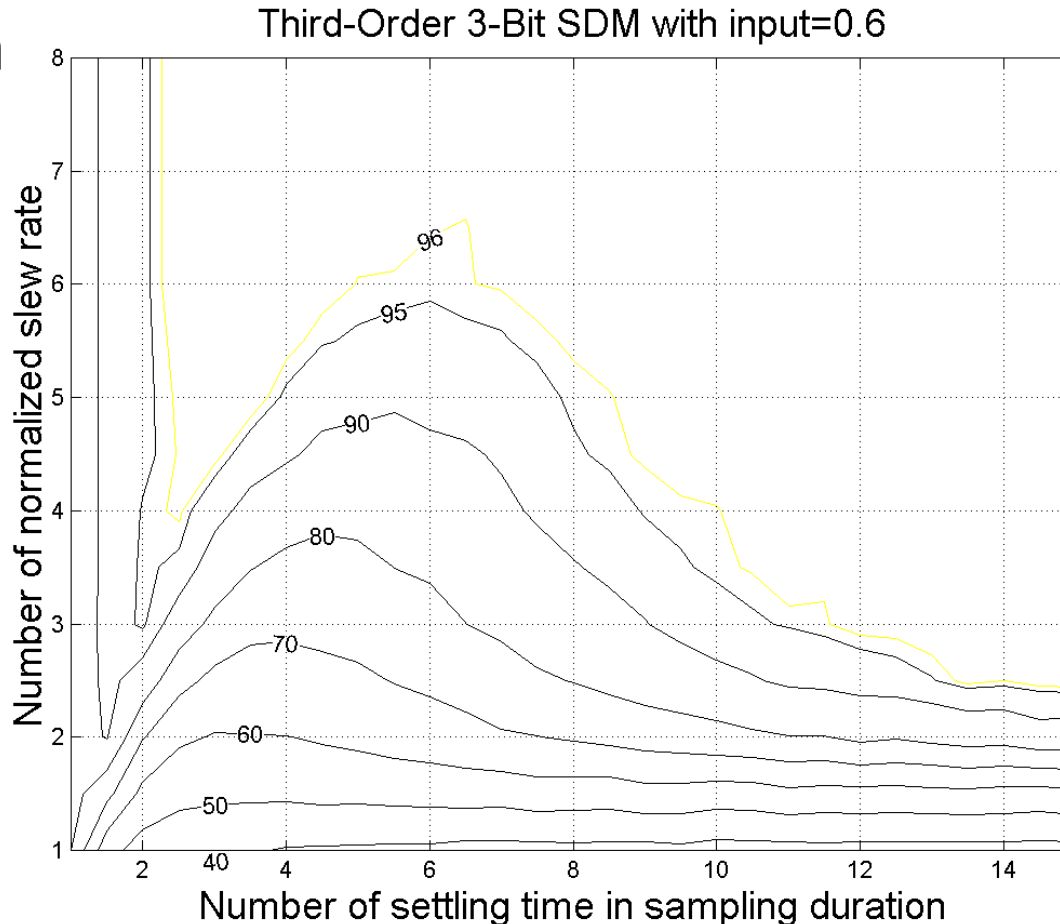
S : slew rate

wt : opamp unit gain frequency

Sn: normalized slew rate

# Opamp Bandwidth and Slew Rate(cont.)

- Simulation



- Choose  $SR=6*SR_n$ ,  
 $T_s=7*t$ ,  $SR=6*2.134*10^6=12.8V/\mu$ ,  $t=T_s/7=55.8nS$ ,  $f_t=$   
 $\omega t/(2\pi)=11.2MHz$

# Capacitance Nonlinearity

- Capacitance value is dependent on the voltage drop of the capacitor

$$C(v) = C_0(1 + k_1v + k_2v^2)$$

- Integrator output

$$v_o(n) - v_o(n-1) \cong g \left\{ v_i(n-1) - \frac{k_1}{2} v_i(n-1) [-v_i(n-1) + v_o(n) + v_o(n-1)] \right. \\ \left. - \frac{k_2}{3} v_i(n-1) [-v_i^2(n-1) + v_o^2(n) + v_o(n)v_o(n-1) + v_o^2(n-1)] \right\}$$

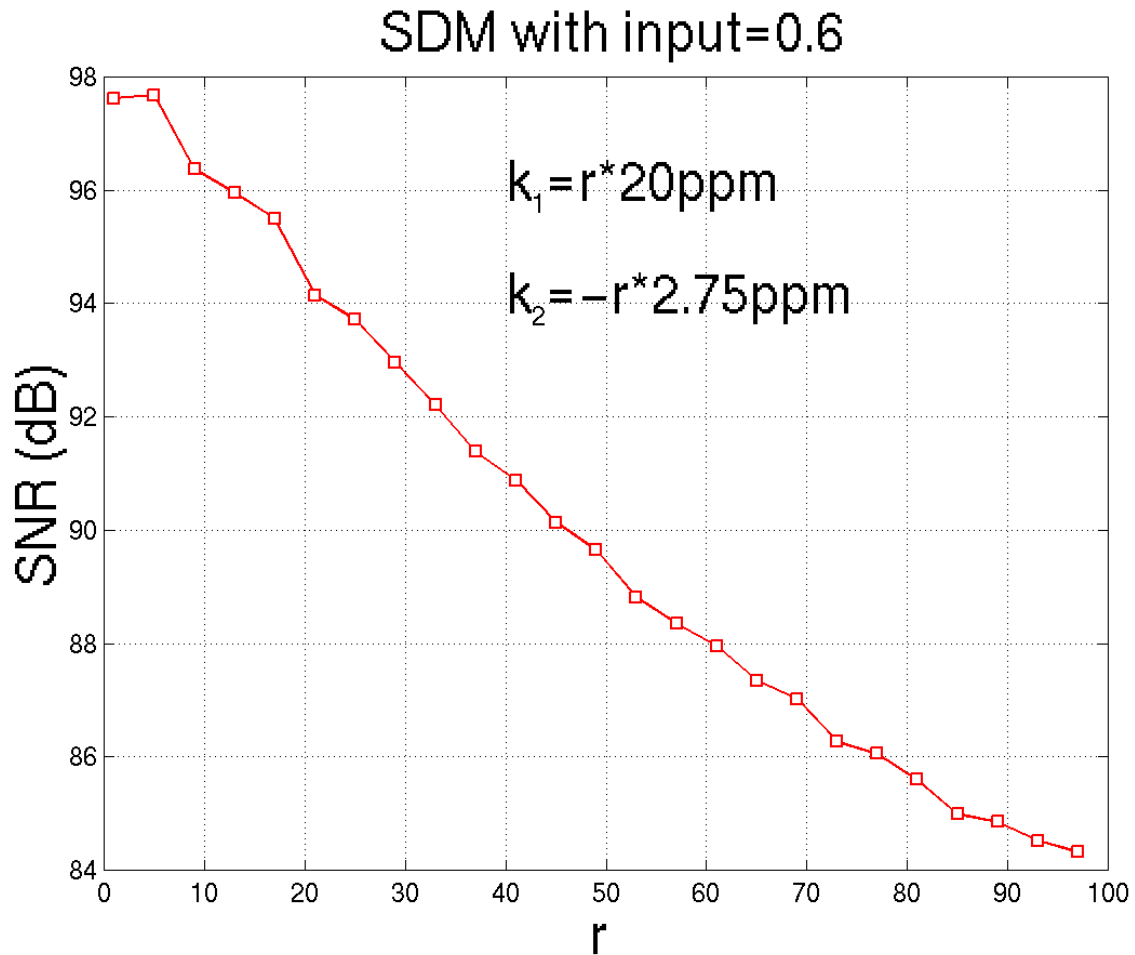
- Harmonic distortion

$$HD_2 = \frac{k_1}{2} \sqrt{v_o^2 + \left(\frac{v_i}{2}\right)^2}$$

$$HD_3 = \frac{k_2}{2} \sqrt{3v_o^2 + \left(\frac{v_i}{2}\right)^2}$$

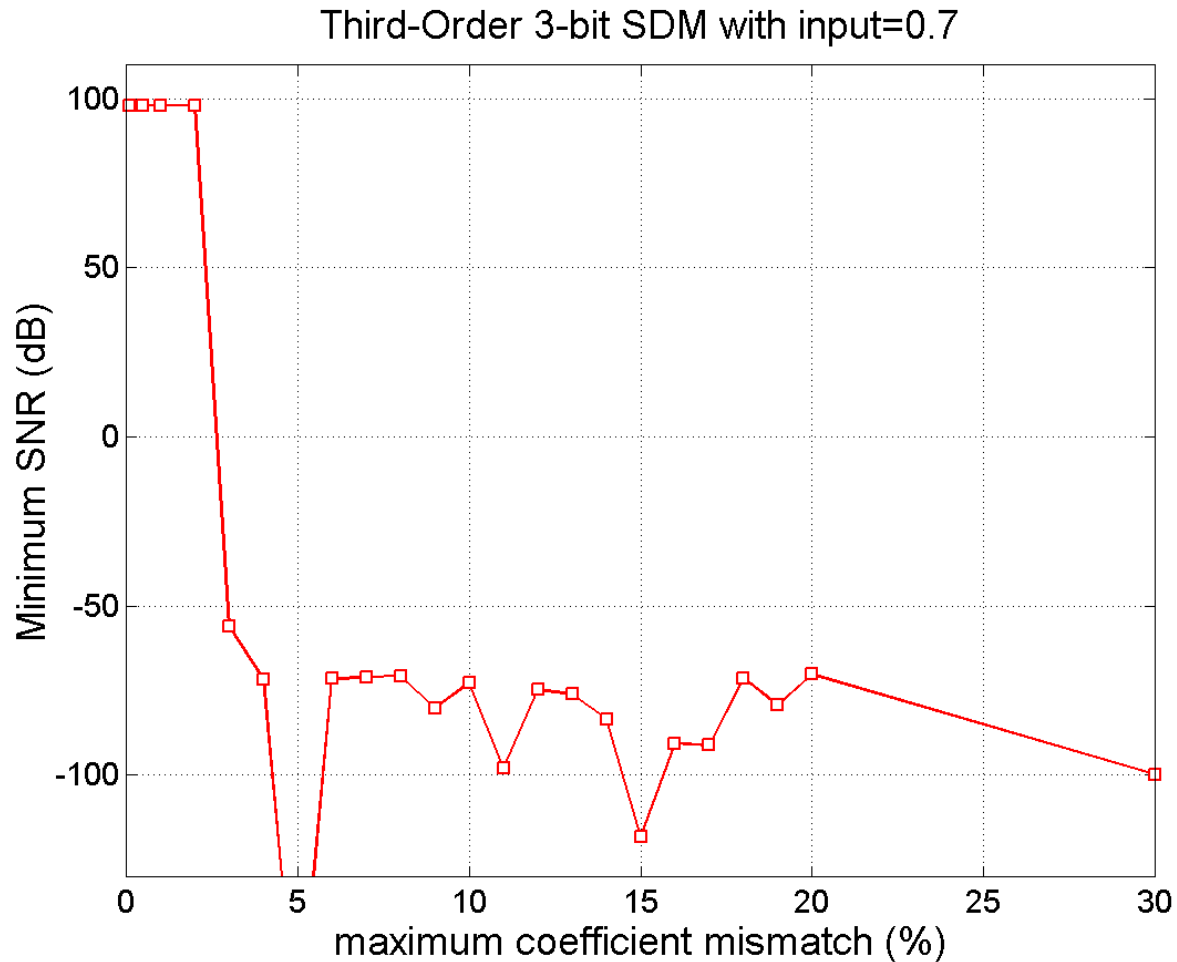
# Capacitance Nonlinearity(cont.)

- Simulation



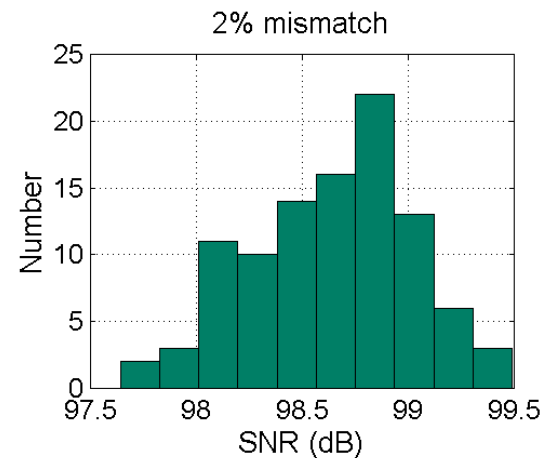
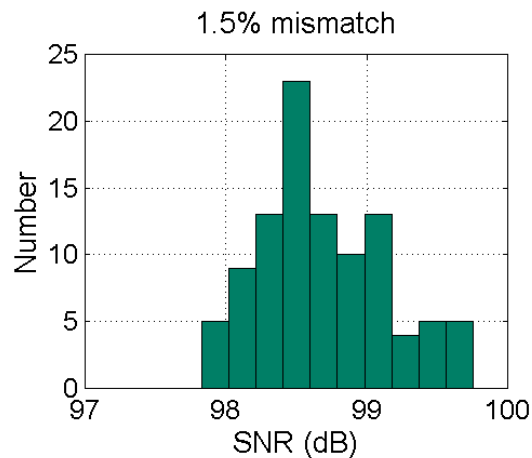
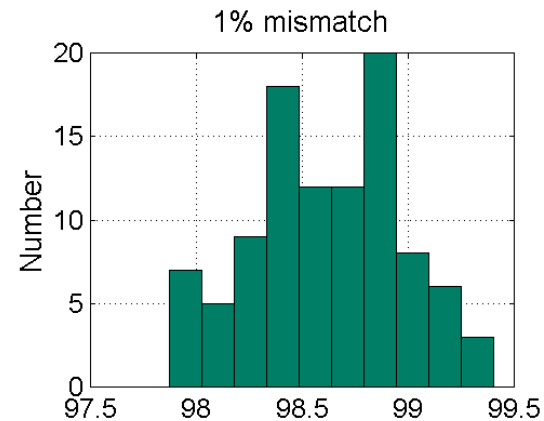
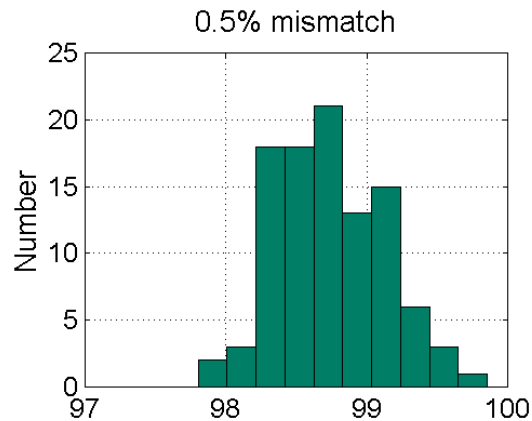
# Coefficient Mismatch

- Coefficient mismatch with uniform random distribution (100 iterations)



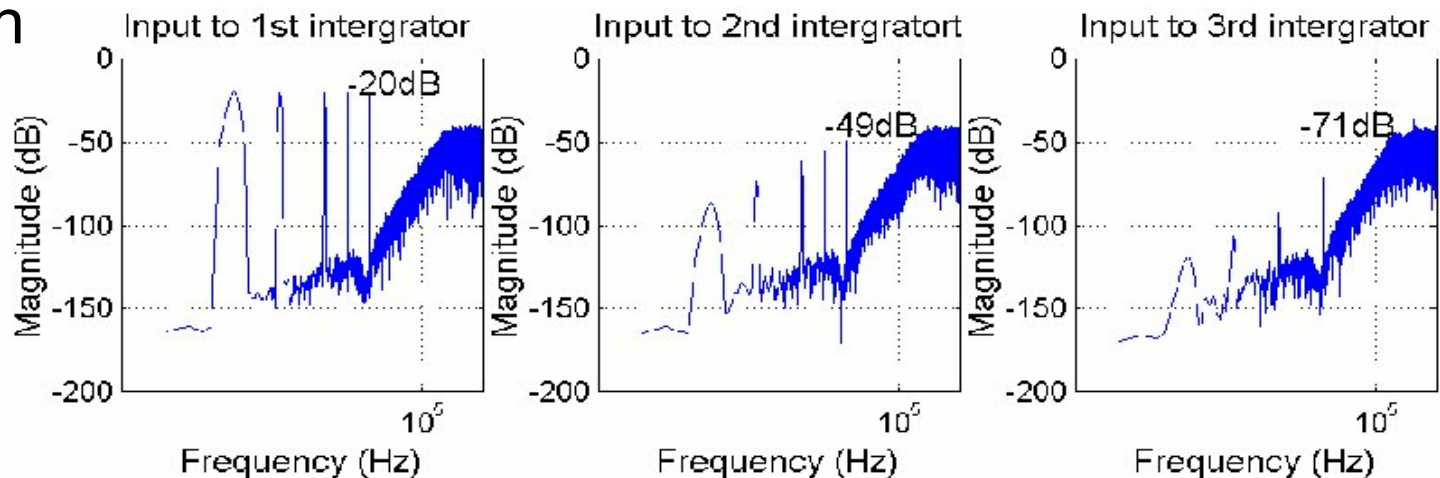
# Coefficient Mismatch(conti.)

- SNR performance of the SDM with coefficient mismatch



# Noise Budget

- Noise source in the baseband
  - Quantization noise (-101dB)
  - Switch noise= $2kT/(OSR*Cs)$
  - Amplifier noise(thermal noise+flicker noise)
  - DAC noise
- Noise budget of third integrators can be fast evaluated by SDM output spectrum while injecting the same signals to the input of each integrator
- Simulation





## Noise Budget(conti.)

- Baseband quantization noise=-101dB
- For the 1st-stage integrator
  - Switch noise  $\leq -104\text{dB}$
  - Amplifier noise  $\leq -104\text{dB}$
  - $C_{s1} \geq 3.5\text{pF}$
- For the 2nd-stage integrator
  - Switch noise  $\leq -85\text{dB}$
  - Amplifier noise  $\leq -85\text{dB}$
  - $C_{s2} \geq 164\text{fF}$
- For the 3rd-stage integrator
  - Switch noise  $\leq -63\text{dB}$
  - Amplifier noise  $\leq -63\text{dB}$
  - $C_{s3} \geq 1.6\text{fF}$

# DAC Noise

- Component mismatch with uniform distribution (50 iteration)

