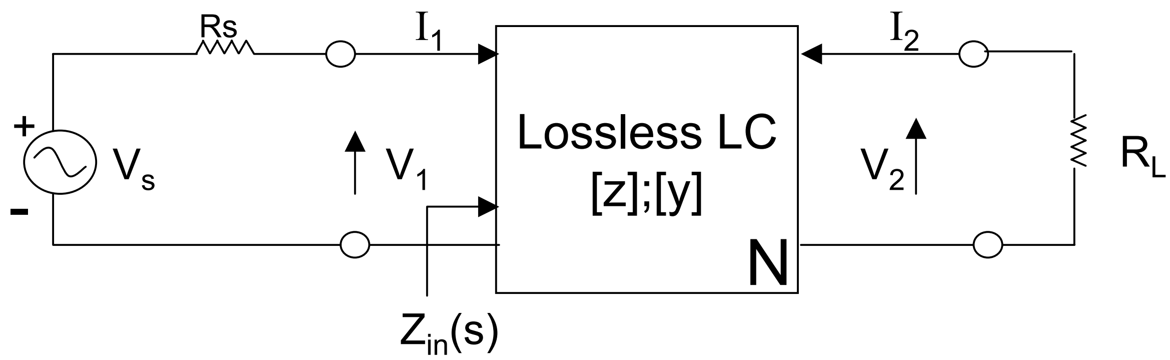
The Design of LC Filters

- Conventional
  — two ports LC (lossless), four-terminal single-input single-output
  a. Rapid changes of magnitude and phase v.s. frequency
  b. Steep slope between passbands and stopbands
  c. Lossless, i.e., dissipates no power itself
- LC lossless still important for high frequency application
- Understanding LC provides us some benefits:
  1. Intuitive understanding of filter concept
  2. Many active RC and SC designs are based on active simulation of passive LC filters
  3. Very low passband sensitivities of lossless LC ladder filters to element tolerances

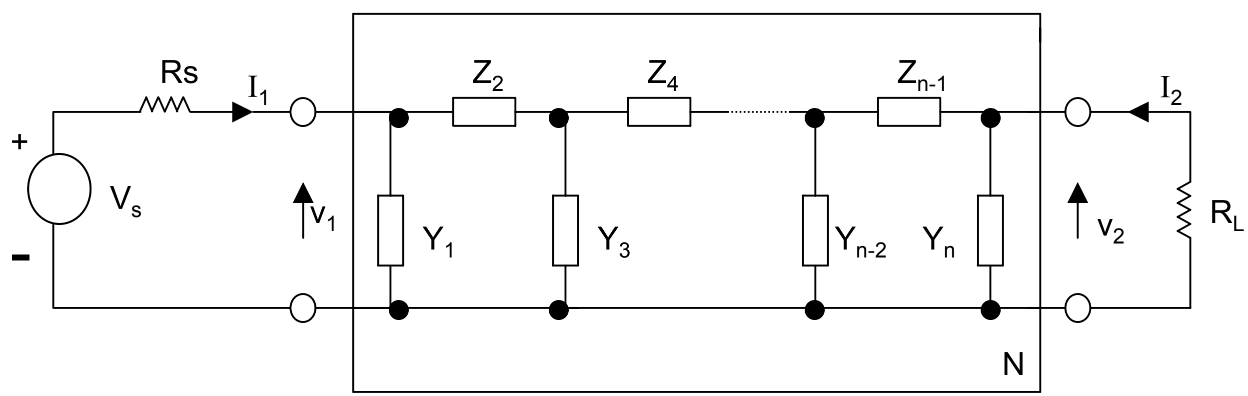
The Design of LC Filters(cont.)

- Lossless LC contains only L&C

— Resistively terminated lossless twoport

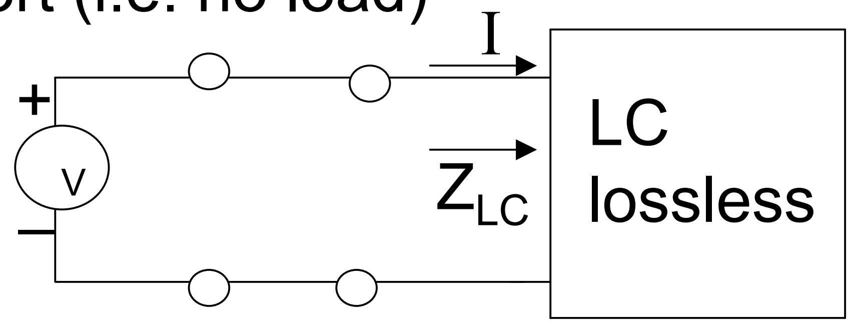


— Resistively terminated ladder structure



Realization of LC Immittance Functions

- Immittance : collective name for impedance and admittance
- LC one port (i.e. no load)



- Lossless means no power consumption

$\Rightarrow P=|I(j\omega)^2|Re\{Z_{LC}(j\omega)\}=0$

$\Rightarrow Re\{Z_{LC}(j\ \omega)\}=0$

$\Rightarrow Z_{LC}(s)=\frac{N(s)}{D(s)}$ is an odd rational function of s

$$=K\frac{(S^2+w_1^2)(S^2+w_3^2)......(S^2+w_{2n+1}^2)}{S(S^2+w_2^2)(S^2+w_4^2).....(S^2+w_{2n}^2)}$$

Most general form degree 2n+1

where K is a positive constant

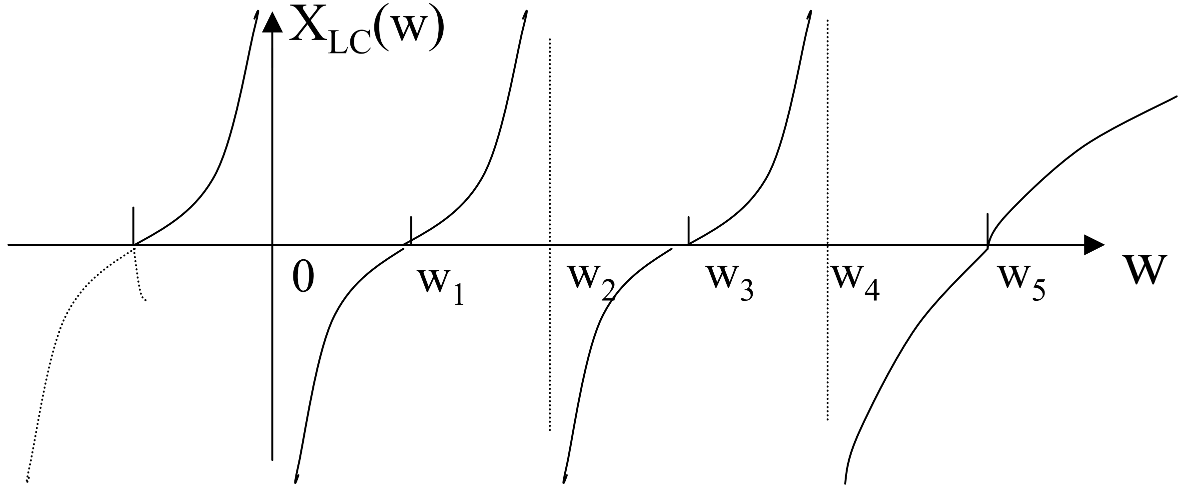
$0\le\omega_1<\omega_2<\omega_3<\bullet\bullet\bullet\bullet\ \omega_{2n}<\omega_{2n+1}<\infty$

Realization of LC Immittance Functions(cont.)

poles and zeros are alternate on the $j\omega$-axis

at zeros $Z_{Lc}(j\omega_{2i+1})=0$ at poles $Z_{Lc}(j\omega_{2k})=\infty$

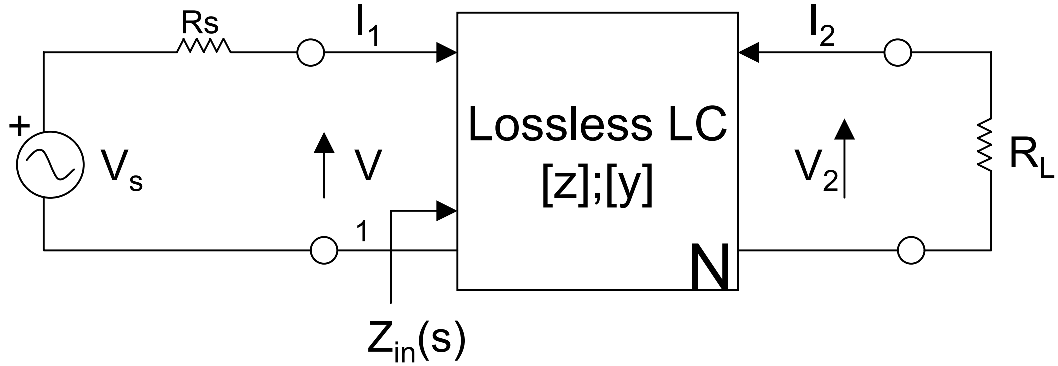
— Example: n=2 is shown below



- All realizations of LC oneports are based on consecutive removal of poles of $Z_{LC}(j\omega)$or $Y_{LC}(j\omega)$
- Example 2-1 ~ 2-4
  — lossless LC networks can be realized as many different prescribed forms

Derivation of Twoport Parameters of LC Filters

- Take into account R_s and R_L



— P_1: power delivered by source to the twoport input

P_2: power delivered to the load R_L

$$P_1 = |I_1(j\omega)|^2 \mathrm{Re}\{Z_{in}(j\omega)\} = \frac{|Vs|^2}{|Rs + Zin(jw)|^2} \mathrm{Re}\{Z_{in}(j\omega)\} \ldots\ldots (1)$$

$$P_2 = \frac{|V_2|^2}{R_L} \ldots\ldots (2)$$

($P_1 = P_2$ because LC is lossless)

— $P_{1(max)}$ occurs when $Z_{in}(j\omega) = R_s$

=> $P_{1(max)} = P_{2(max)} = \frac{1}{4}\frac{|Vs|^2}{R_s}$

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Derivation of Twoport Parameters of LC Filters(cont.)

— Define transfer function

$$|H(j\omega)|^2 = \frac{P_2}{P_{1(max)}} = \frac{4R_s}{R_L}\left|\frac{V_2}{V_s}\right|^2$$

$$\Rightarrow H(s) = \sqrt{\frac{4R_s}{R_L}}\frac{V_2}{V_s} = \frac{N(s)}{D(s)}$$

$(|H(j\omega)| \le 1$ since $P_2 = P_1 \le P_{1(max)})$

— Equating Eqs. (1) and (2) yields

$$|H(j\omega)|^2 = \frac{4R_s \, Re\{Z_{in}(j\omega)\}}{|R_s + Z_{in}(j\omega)|^2} = 1 - \left|\frac{R_s - Z_{in}(j\omega)}{R_s + Z_{in}(j\omega)}\right| = 1 - |\rho(j\omega)|^2$$

where $\rho(s) = \pm\frac{R_s - Z_{in}(s)}{R_s + Z_{in}(s)}$ is reflection coefficient

— $P_1 + P_r = P_{1(max)}$ where $P_r = P_{1(max)} |\rho(j\omega)|^2$

— $|\rho(j\omega)|^2 = 1 - |H(j\omega)|^2 = \frac{|D(jw)^2 - |N(jw)|^2}{|D(jw)|^2} = \varepsilon^2 \frac{|F(j\omega)|^2}{|D(j\omega)|^2}$

$$\Rightarrow \rho(s) = \pm\varepsilon\frac{F(s)}{D(s)} = \pm\frac{\bar{F}(s)}{D(s)}$$

Derivation of Twoport Parameters of LC Filters(cont.)

where $\overline{F}$ is reflection zero polynomial

N(s) is transmission zero polynomial

D(s) is natural frequency polynomial

$$\Rightarrow Z_{in}(s) = R_s \frac{D(s) \mp \overline{F}(s)}{D(s) \pm \overline{F}(s)} ==> \frac{Z_{in}}{R_s} = \frac{1 \mp \rho(s)}{1 \pm \rho(s)} \quad \text{.......................(Eq2-12)}$$

dual network, i.e., either upper or low sign can be chosen

—Twoport parameters

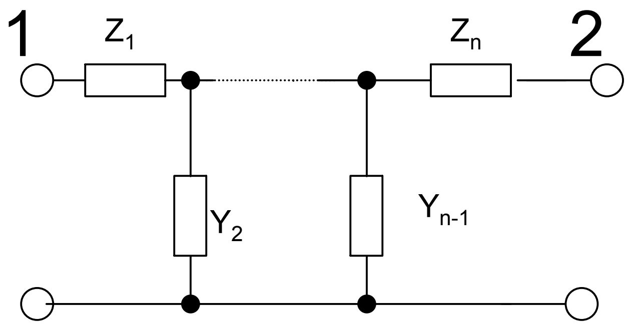
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \text{ and } \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

where $Z_{12} = Z_{21}$ and $Y_{12} = Y_{21}$ for passive reciprocal network

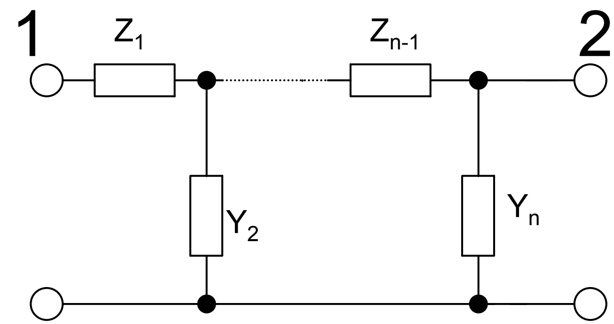
Derivation of Twoport Parameters of LC Filters(cont.)

— Possible input and output configurations of ladders.

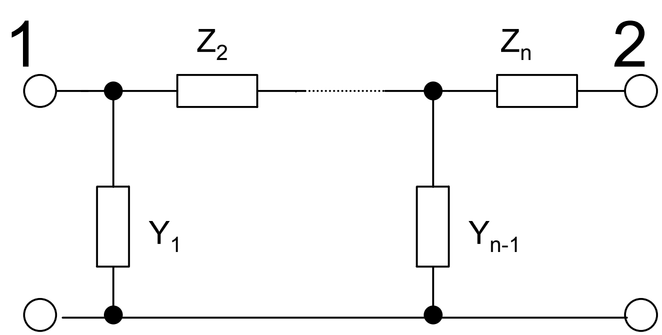
(a)Use Y_{11} or Y_{22}



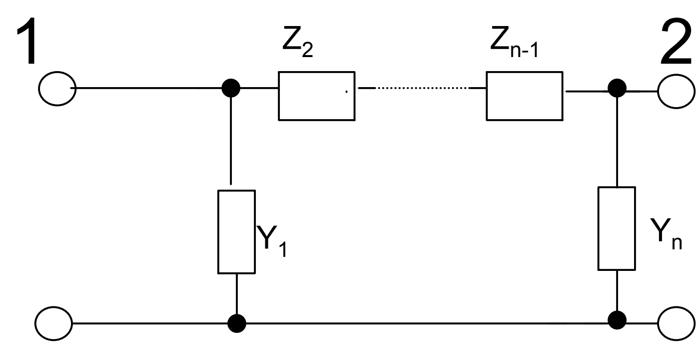
(b)Use Z_{11} or $Y_{22;}$



(c)Use Y_{11} or $Z_{22;}$



(d)Use Z_{11} orZ_{22}

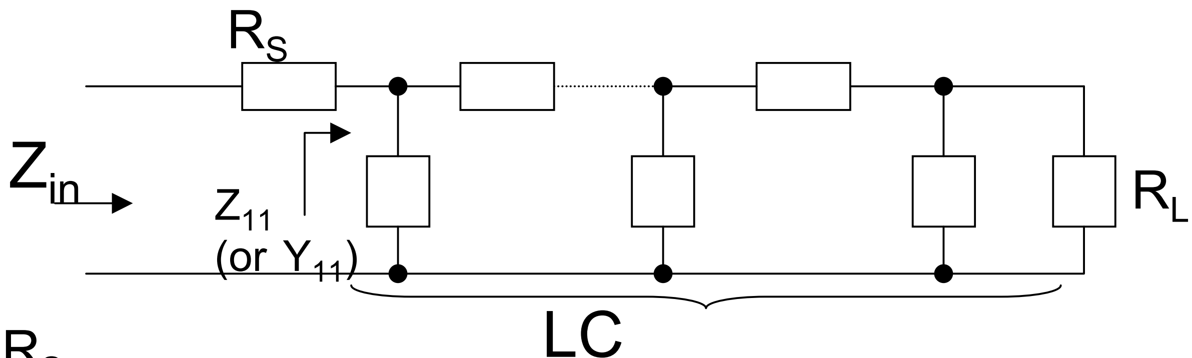


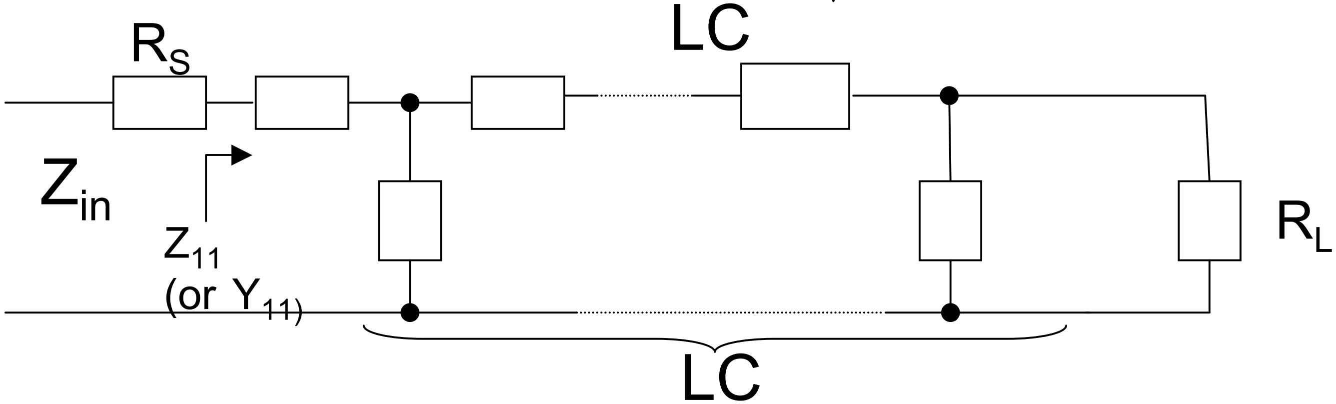
Derivation of Twoport Parameters of LC Filters(cont.)

$$H(s)=\frac{\sqrt{4R_sR_L}Z_{12}}{(Z_{11}+R_s)(Z_{22}+R_L)-Z_{12}^2}=\frac{N(s)}{D(s)}$$

$$Z_{in}=\frac{Z_{11}R_L+Z_{11}Z_{22}-Z_{12}^2}{Z_{22}+R_L} \quad \text{.............................. (2-17)}$$

- Example





- Let $R_s\frac{D(s)\mp\bar{F}(s)}{D(s)\pm\bar{F}(s)}=\frac{Z_{11}R_L+Z_{11}Z_{22}-Z_{12}^2}{Z_{22}^2+R_L}$ => Darlington procedure

Eq. (2-12) Eq.(2-17)

Derivation of Twoport Parameters of LC Filters(cont.)

- Darlington Procedure :

THE z- AND y-PARAMETERS OF AN LC FILTER WITH TRANSMISSION ZERO POLYNOMIAL N(S), REFLECTION ZERO POLYNOMIAL $\overline{F}$(S), AND NATURAL FREQUENCY POLYNOMIAL D(S)

N(s)	z_{11}/R_s	z_{22}/R_L	$Z_{12}/\sqrt{R_s R_L}$	$Y_{11}R_s$	$Y_{22}R_L$	$= Y_{12}\sqrt{R_S R_L}$
Even	m_1/n_2	m_2/n_2	N/n_2	m_2/n_1	m_1/n_1	N/n_1
Odd	n_1/m_2	n_2/m_2	N/m_2	n_2/m_1	n_1/m_1	N/m_1

The even and odd polynomials $m_i(s)$ and $n_i(s)$, i=1, 2, are obtained from $Z_{in}(s)=(m_1+n_1)/(m_2+n_2)$ and are defined as

$m_1(s) = D_e(s) - \overline{F_e}(s)$ $\quad n_1(s) = D_0(s) - \overline{F_o}(s)$ $\quad N = \sqrt{m_1 m_2 - n_1 n_2}$

$m_2(s) = D_e(s) + \overline{F_e}(s)$ $\quad n_2(s) = D_0(s) + \overline{F_o}(s)$

$$D(s) = D_e(s) + D_0(s) \triangleq Ev\{D(s)\} + Od\{D(s)\}$$

$$\overline{F_o}(s) = \overline{F_e}(s) + \overline{F_o}(s) \triangleq Ev\{\overline{F}(s)\} + Od\{\overline{F}(s)\}$$

Realization of LC Ladders

- Example : 2dB-ripple Chebyshev Filter(5th-order)

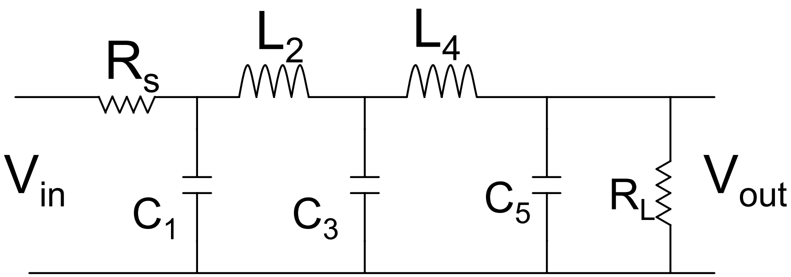


Fig. a

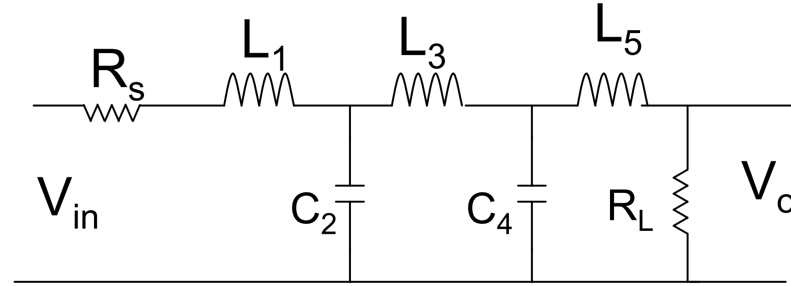


Fig. b

(a)

$$H(s)=\frac{0.0817}{s^5+0.7065s^4+1.4995s^3+0.6935s^2+0.4593s+0.0817}=\frac{N(s)}{D(s)}$$

Find reflection function $\rho(s)$

$$|\rho(\omega)|^2=\frac{|D(j\omega)|^2-|N(j\omega)|^2}{|D(j\omega)|^2}=\left|\frac{\bar{F}(s)}{D(s)}\right|^2$$

$$\rho(s)=\pm\frac{\bar{F}(s)}{D(s)}=\pm\frac{s^5+1.25s^3+0.3125s}{D(s)}$$

$$=\pm\frac{s^5+1.25s^3+0.3125s}{s^5+0.7065s^4+1.4995s^3+0.6935s^2+0.4593s+0.0817}$$

Realization of LC Ladders(cont.)

(i) Fig. a (select upper sign of $\rho(s) = \pm\frac{\bar{F}(s)}{D(s)}$)

$m_1=m_2=0.7065s^4+0.6935s^2+0.0817$

$n_1=0.2495s^3+0.1468s$

$n_2=2s^5+2.7495s^3+0.7718s$

$$Z_{11}=Z_{22}=\frac{m_1}{n_2}=\frac{0.7065s^4+0.6935s^2+0.0817}{2s^5+2.7495s^3+0.7718s}$$

$$Y_{11}=\frac{1}{Z_{11}}=\frac{2s^5+2.7495s^3+0.7718s}{0.7065s^4\,0.6935s^2+0.0817}$$

$$=2.831s+\cfrac{1}{0.899s+\cfrac{1}{3.782s+\cfrac{1}{0.899s+\cfrac{1}{2.831s}}}}$$

$\therefore$ by normalized circuit with BW=1 & $R_s=R_L=1$

$C_1=C_5=2.831F$, $C_3=3.782F$, $L_2=L_4=0.899H$

Realization of LC Ladders(cont.)

(ii) Fig. b (select lower sign)

$m_1=m_2=0.7065s^4+0.6935s^2+0.0817$

$n_1=2s^5+2.7495s^3+0.7718s$

$n_2=0.2495s^3+0.1468s$

$$Y_{11}=Y_{22}=\frac{m_1}{n_1}=\frac{0.7065s^4+0.6935s^2+0.0817}{2s^5+2.7495s^3+0.7718s}$$

$$Z_{11}=\frac{1}{Y_{11}}=2.831s+\cfrac{1}{0.899s+\cfrac{1}{3.782s+\cfrac{1}{0.899s+\cfrac{1}{2.831s}}}}$$

$\therefore$ by normalized circuit with BW=1 & $R_S=R_L=1$

$L_1=L_5=2.831H$, $L_3=3.782H$, $C_2=C_4=0.899F$