Sensitivity

- Real components deviate from their designed values
  - Initially inaccurate due to fabrication tolerances.
  - drift due to environmental effects such as temperature and humidity.
  - chemical changes which occurs as the circuit ages
  - inaccuracies in modeling the passive and active devices, e.g.,nonideal OPAMP and parasitics.
- All coefficients, and therefore poles and zeros of H(s), depend on circuit element.
- The size of H(s) error depends on how large the component tolerances are and how sensitive the circuit's performance is tothese tolerances

Sensitivity(Cont.)

- Sensitivity calculation allows the designer
  - to select the better circuit from those in the literature.
  - to determine whether a chosen filter circuit satisfies and will keep satisfying the given specifications.
- Component x
- Performance criterion P(x), such as
  - quality factor
  - pole frequency
  - zero frequency

  or P(s,x), if P is also a function of frequency and stands for
  - H(s), or
  - magnitude of H(s), or
  - phase of H(s)

Sensitivity(Cont.)

- Sensitivity S_x^p
  - –Taylor series

$$P(s,x) = P(s,\ x_0) + \left.\frac{\partial P(s,x)}{\partial x}\right|_{x_0} dx + \frac{1}{2}\left.\frac{\partial^2 P(s,x)}{\partial x^2}\right|_{x_0} (dx)^2 + \cdots$$

$$\text{if } \frac{dx}{x_0} << 1 \text{ and } \left.\frac{dP}{dx}\right|_{x=x_0} \text{ is small}$$

$$\Rightarrow \begin{cases} \Delta P(s,\ x_0) = P(s, x_0 + dx) - P(s, x_0) \approx \left.\dfrac{\partial P(s,x)}{\partial x}\right|_{x_0} dx \\[2ex] \dfrac{\Delta P(s,\ x_0)}{P(s,\ x_0)} \approx \dfrac{x_0}{P(s,\ x_0)} \left.\dfrac{\partial P(s,x)}{\partial x}\right|_{x_0} \dfrac{dx}{x_0} \end{cases}$$

$$\Rightarrow S_x^P = \frac{x_0}{P(s,\ x_0)} \left.\frac{\partial P(s,x)}{\partial x}\right|_{x_0} = \left.\frac{\partial P / P}{\partial x / x}\right|_{x_0} = \left.\frac{d(\ln P)}{d(\ln x)}\right|_{x_0}$$

$$\text{if } \frac{\Delta x}{x_0} << 1, \text{ then } S_x^P \approx \frac{\Delta P / P}{\Delta x / x}$$

Sensitivity(Cont.)

- Important points:

1. What is more important in the final circuit performance?

$$\frac{\Delta P}{P} \text{ instead of } S_x^p$$

$\Rightarrow$ acceptable design

$$\text{large } S_x^p \text{ but } \frac{\Delta P}{P} \approx 0$$

$$\text{or large } \frac{\Delta x}{x} \text{ but } \frac{\Delta P}{P} \approx 0$$

$\Rightarrow$ useless design

$$\text{small } S_x^p \text{ but large } \frac{\Delta P}{P}$$

$$\text{or small } \frac{\Delta x}{x} \text{ but large } \frac{\Delta P}{P}$$

Sensitivity(Cont.)

2. Good circuits should have small sensitivities to their elements.
3. For acceptable performance deviations(variabilities) $\frac{\Delta P}{P}$,they can be assembled from elements having larger tolerances $\frac{dx}{x}$ and they are, therefore, more economical to build. Also, if their components vary during operation, they are less likely to drift out of the acceptable region of specifications (i.e. the acceptable region).

- Example 3-1

Single-parameter Sensitivity

- The smallest and largest single-parameter sensitivities are of practical importance because they identify the least and most critical circuit components
  - the most precise component for the largest sensitivity
  - cheaper component for the smaller sensitivity
- Many multi-parameter sensitivity measures are conveniently expressed in terms of single-parameter sensitivities.
- Sensitivity
  - Single-parameter sensitivity, $S_x^{P(x)}$

$$S_x^{P(x)} = \frac{x}{p}\frac{dP}{dx} = \frac{d(\ln P)}{d(\ln x)}$$

  - Semirelative sensitivity measure

$$Q_x^{P(x)} = x\frac{dP}{dx} = PS_x^{P(x)}$$

Single-parameter Sensitivity(Cont.)

– Important properties

$$S_x^{P1P2} = S_x^{P1} + S_x^{P2} \; ; \text{ where } P(x) = P_1(x)P_2(x)$$

$$S_x^{P1/P2} = S_x^{P1} - S_x^{P2} \; ; \text{ where } P(x) = \frac{P_1(x)}{P_2(x)}$$

$$S_x^{P(y(x))} = S_y^{P}\, S_x^{y} \; ; \text{ where } P = P(y) \;\&\; y = y(x)$$

$$S_T^{Wz} = S_R^{Wz}\, S_T^{R} + S_C^{Wz}\, S_T^{C} \; ; \text{ where zero frequency } Wz = Wz(R, C)$$

$$R = R(T)$$

$$C = C(T)$$

Sensitivity Invariants

- Homogeneity condition

$$P(kx_1, kx_2, \cdots, kx_n) = k^{\lambda} P(x_1, x_2, \cdots, x_n)$$

$$\Rightarrow \frac{\partial}{\partial k}\left[P(kx_1, kx_2, \cdots, kx_n)\right] = \frac{\partial}{\partial k}\left[k^{\lambda} P(x_1, x_2, \cdots, x_n)\right]$$

$$\Rightarrow \sum_{i=1}^{n} x_i \frac{\partial P}{\partial (kx_i)} = \lambda k^{\lambda-1} P \ \ ; \text{ where } \lambda \text{ is an integer constant}$$

Dividing both sides by P and setting $k = 1$

$$\Rightarrow \sum_{i=1}^{n} S_{x_i}^{P} = \lambda \ \ \text{----Euler's formula}$$

- Homogeneity condition (or Euler's formula) can be applied on filter scaling

1. Impedance scaling

Transfer function $H(s) = H(S, R_i, C_i, L_i, \mu_i, \alpha_i, g_{mi}, r_{mi})$

$$\text{where } \mu = \frac{V_{out}}{V_{in}}, \alpha = \frac{I_{out}}{I_{in}}, g_m = \frac{I_{out}}{V_{in}}, r_m = \frac{V_{out}}{I_{in}}$$

Sensitivity Invariants(Cont.)

$$\Rightarrow H(S, \frac{R_i}{R_o}, R_o C_i, \frac{L_i}{R_o}, \mu_i, \alpha_i, R_o g_{mi}, \frac{r_{mi}}{R_o}) = H(s)$$

because H is not changed by any impedance scaling

$$\Rightarrow \lambda = 0$$

$$\Rightarrow \sum_i S^H_{Ri} - \sum_i S^H_{Ci} + \sum_i S^H_{Li} - \sum_i S^H_{gmi} + \sum_i S^H_{rmi} = 0$$

2. Frequency scaling

$$H(S, R_i, \frac{L_i}{W_n}, \frac{C_i}{W_n}, \mu_i, \alpha_i, g_{mi}, r_{mi}) = H(\frac{S}{W_n})$$

$$\frac{\partial}{\partial\left(\frac{1}{W_n}\right)}\left[H(S, R_i, \frac{L_i}{W_n}, \frac{C_i}{W_n}, \mu_i, \alpha_i, g_{mi}, r_{mi})\right] = \frac{\partial}{\partial\left(\frac{1}{W_n}\right)} H(\frac{S}{W_n})$$

Setting $W_n = 1$ and dividing by H

$$\Rightarrow \sum_i S^H_{Li} + \sum_i S^H_{Ci} = S^H_S$$

Sensitivity Invariants(Cont.)

– Examples:

$$1.\ \text{LC filters}: \begin{cases} \sum_i S_{Li}^H - \sum_i S_{Cj}^H = -\sum_{k=1}^{2} S_{Rk}^H \\ \sum_i S_{Li}^H + \sum_j S_{Cj}^H = S_S^H \end{cases}$$

; where R1 and R2 are the source and load resistors

$$2.\ \text{RC filters}: \begin{cases} \sum_i S_{Ri}^H - \sum_j S_{Cj}^H = 0 \\ \sum_j S_{Cj}^H = S_S^H \end{cases}$$

- Example 3-2

High-order active filters

- Recall that $$S_x^H = \frac{x}{N(s)D(s)}\left[D(s)\frac{\partial N(s,x)}{\partial x} - N(s)\frac{\partial D(s,x)}{\partial x}\right]$$
$$= S_x^k - \sum_{i=1}^{m}\frac{Q_x^{Zi}}{S - Zi} + \sum_{i=1}^{n}\frac{Q_x^{Pi}}{S - Pi}$$

=> 1. S_x^H Is quite large in the neighborhood of any pole or zero because S_x^H has poles at all poles and zeros of H(s)

2. passband, especially near passband edges, sensitivities is large because many or all of the filter poles are distributed over the passband close to the jw-axis.

=> direct realization of a high-order transfer function will in general lead to unacceptable variability dH/H due to dx/x.

High-order active filters(Cont.)

=> Practical design approaches:

1. Cascade design based on biquad

   any component x affects only one pole pair or zero pair, then S_x^H can be much-reduced.

2. LC ladder simulation

   where $\left[D\frac{\partial N}{\partial x} - N\frac{\partial D}{\partial x} \right]$ of S_x^H can be low in the frequency range of interest.

Cascade design

- Goal

Element x affects only one pole pair and/or zero pair so that the effects of Δx are isolated from all other critical frequencies.

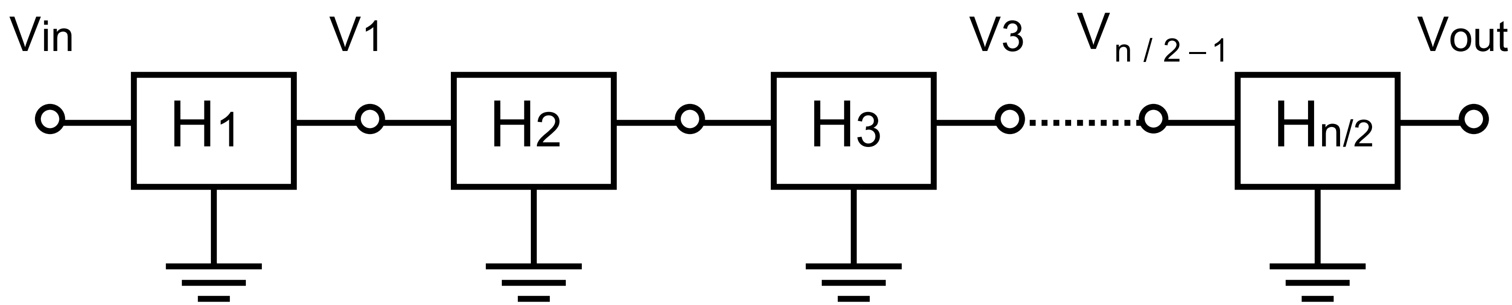
i.e. $$H(s)=\prod_{k=1}^{n/2}H_k(s)=\prod_{k=1}^{n/2}\frac{\alpha_{2k}S^2+\alpha_{1k}S+\alpha_{0k}}{S^2+S\frac{\omega_{pk}}{Q_{pk}}+\omega_{pk}^2}$$; if n is assumed to be even

where $H_k(s)$ can be realized as a biquad with no interaction between sections

$$H_k(s)=\frac{\alpha_{2k}S^2+\alpha_{1k}S+\alpha_{0k}}{S^2+S\frac{\omega_{pk}}{Q_{pk}}+\omega_{pk}^2}$$ depends on x only a single biquad

Cascade design(Cont.)

- Circuit structure of cascade



$$H(s) = \frac{V_{out}}{V_{in}} = \frac{V_1}{V_{in}} \frac{V_2}{V_1} \frac{V_3}{V_2} \cdots \frac{V_{n/2-1}}{V_{n/2-2}} \frac{V_{out}}{V_{n/2-1}} = H_1(S) H_2(s) \cdots H_{n/2}(s)$$

- Sensitivities

$$S_{H_k(s)}^{H(S)} = \frac{H_k(s)}{H(s)} \frac{\partial H(S)}{\partial H_k(s)} = \frac{H_k(s)}{H(s)} \frac{H(s)}{H_k(s)} = 1$$

$$S_x^{H(S)} = \sum_{m=1}^{n/2} S_{H_m(s)}^{H(s)} S_x^{H_m(S)} = \sum_{m=1}^{n/2} S_x^{H_m(s)} = S_x^{H_k(S)}$$

Cascade design(Cont.)

- Cascade
  - Sensitivity considerations of cascade are far superior to a direct realization as a single high-order block. (Example 3-8)
  - More modular using largely identical building blocks
  - Easier to tune because the blocks do not interact
  - Generally easier to design

Simulation of LC ladders

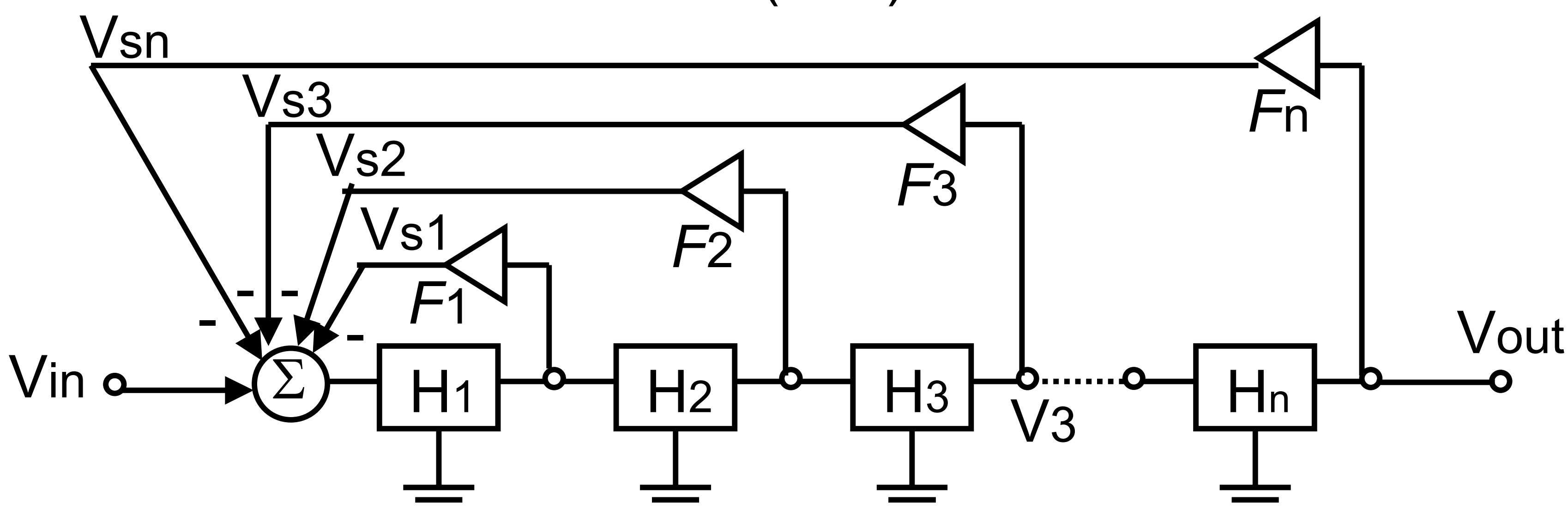
- Excellent sensitivity properties
- Considerable effort for active RC filter to inherit the properties of LC ladders

Multiple-feedback(MF) topologies

- Sensitivity to component variations

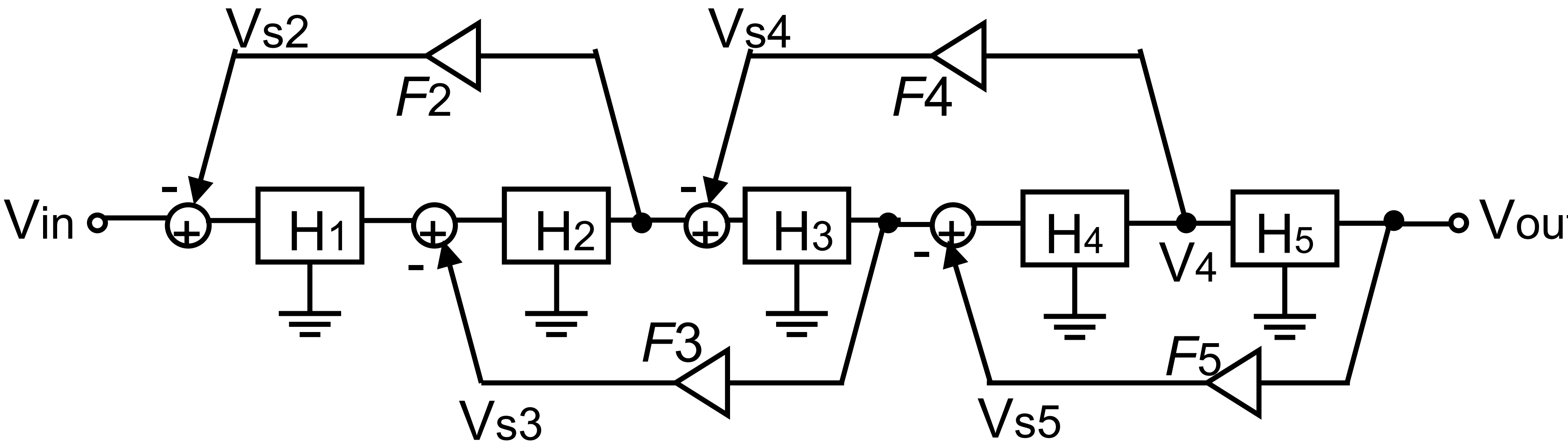
  LC ladder < MF < cascade < direct implementation

  (direct implementation: structures other than LC,cascade, MF, and others)
- MF has lower sensitivity than cascade has higher modularity than LC ladder
- Two particular MF cases:
  - Follow-the-leader feedback (FLF)



Multiple-feedback(MF) topologies(Cont.)

– Leapfrog (LF)



- H(s) = f{Hk(s)}

$S_x^H = S_{Hj}^H S_x^{Hj}$ (in cascade, $S_x^H = S_x^{Hj}$ since $S_{Hj}^H = 1$)

where S_x^H is the sensitivity of H(s) to an element x in section j.

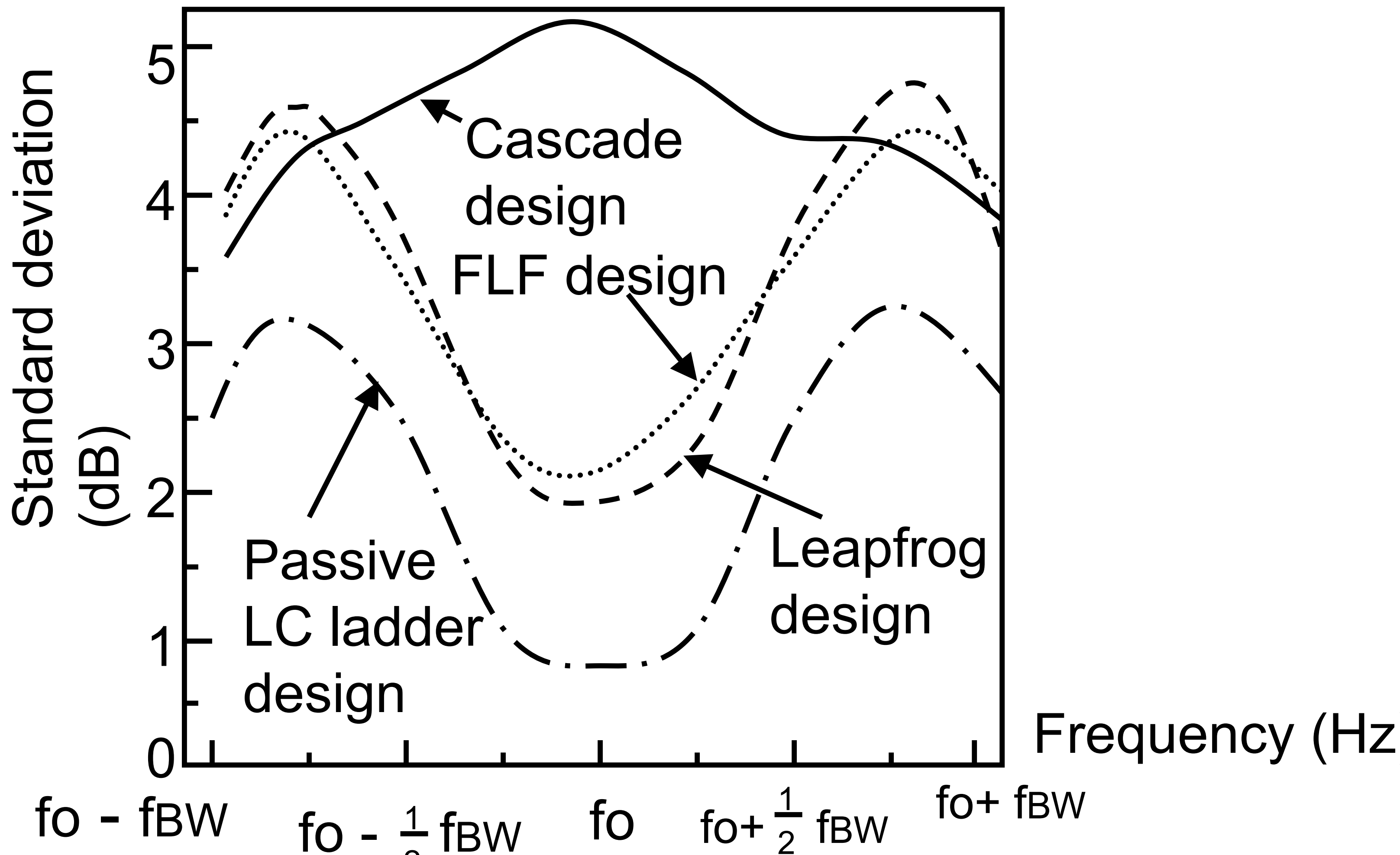
If $S_{Hj}^H < 1$ in the frequency range of interest, then MF design can lead to a better sensitivity behavior better than that of cascade design.

Multiple-feedback(MF) topologies(Cont.)

- Conclusions

1. Feedback paths around low-order sections in an MF filter topology can lead to passband sensitivities lower than those of an equivalent cascade design.
2. The sensitivity improvement is usually largest in the center and becomes less pronounced toward the edges of the passband.
3. In the stopbands, where the feedback paths lose their effectiveness, MF and cascade sensitivities are approximately of the same magnitude.

Comparison



Simulated variations of the sixth-order Butterworth bandpass filter (1.732%passive component tolerances) for different realizations.

Multiparameter Sensitivity

- Usually involve computer-aided routines because of the large volume of computations

Design centering

- Find a nominal circuit design that is centered in the acceptability region
- Example 3-11

The Most Frequently Encountered Errors in Sensitivity Comparison

- Forgetting that sensitivity is a small-change concept.
- Using the wrong frequency range.
- Comparing optimized with unoptimized circuit.
- Considering only sensitivities instead of the more important variabilities
- Relying too much on single-parameter sensitivities