Second-order Active Filter(Biquad)

- Biquad transfer function

$$H(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0} = \frac{a_2(s^2 + (w_Z/Q_Z)s + w_Z^2)}{s^2 + (w_P/Q_P)s + w_P^2}$$

- Biquads
  - fundamental for high-order cascade or multiple loop feedback filters
  - best possible

    a, low sensitivity

    b, economical

    c, manufacturable

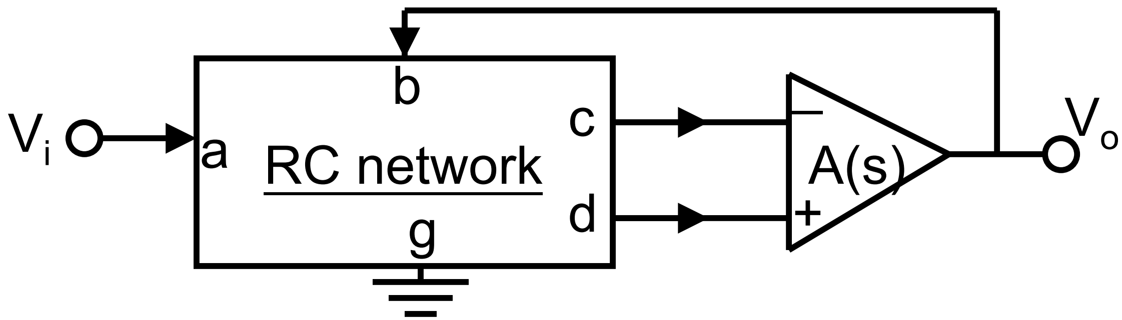
    d, easy to tune

    e, low output resistance (for voltage output)

    f, high input impedance
  - different circuits are claimed by their respective inventors to be advantageous in one way or another

Single-Amplifier Biquad (SAB)

- General feedback configuration



  - transfer function of RC network

$$T_{kl}(s) = \frac{V_k}{V_l} = \frac{N_{kl}(s)}{D_o(s)} \qquad \text{Where } k=c,d \quad l=a,b$$

$$\frac{1}{A(s)} V_o(s) = (T_{da}(s) - T_{ca}(s))V_i(s) + (T_{db}(s) - T_{cb}(s))V_o(s)$$

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{T_{da}(s) - T_{ca}(s)}{T_{cb}(s) - T_{db}(s) + 1/A(s)}$$

  - Second-order RC network is required for 2nd-order H(s)
  - Transmission zeros are determined by the forward path, i.e., $T_{ca}(s)$ and $T_{da}(s)$
  - The poles are set by the feedback paths, i.e., $T_{cb}(s)$and $T_{db}(s)$

Generation of transfer function zeros

- Natural frequencies or poles are found from

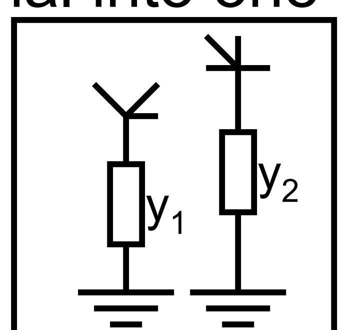
  $T_{cb}(s)-T_{db}(s)+1/A(s)=0$

  =>Poles does not depend on node a, I.e., it is indepent of the connection of input V_i.
- Zeros of H(s) are found from

  $T_{da}(s)-T_{ca}(s)=0$ (depends on node a)

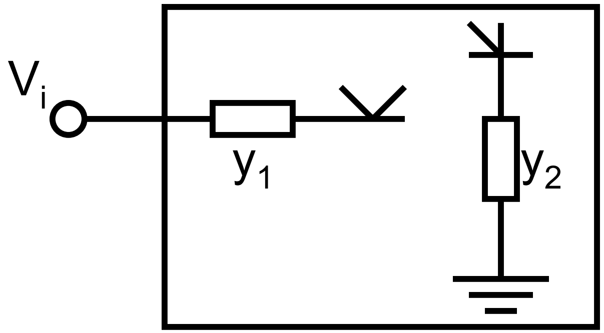
  =>Zeros depend on where the input signal is applied.

  =>The zeros of a transfer function are created without disturbing the poles, by feeding the input signal into any node or nodes that were previously connected to ground.
- Example : by feeding input signal into one node
  - Two ground elements

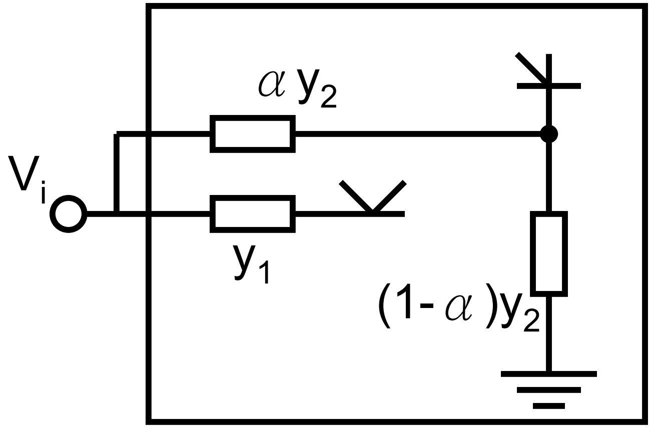


Generation of transfer function zeros(cont.)

– One element is completely lifted off ground and connected to input.

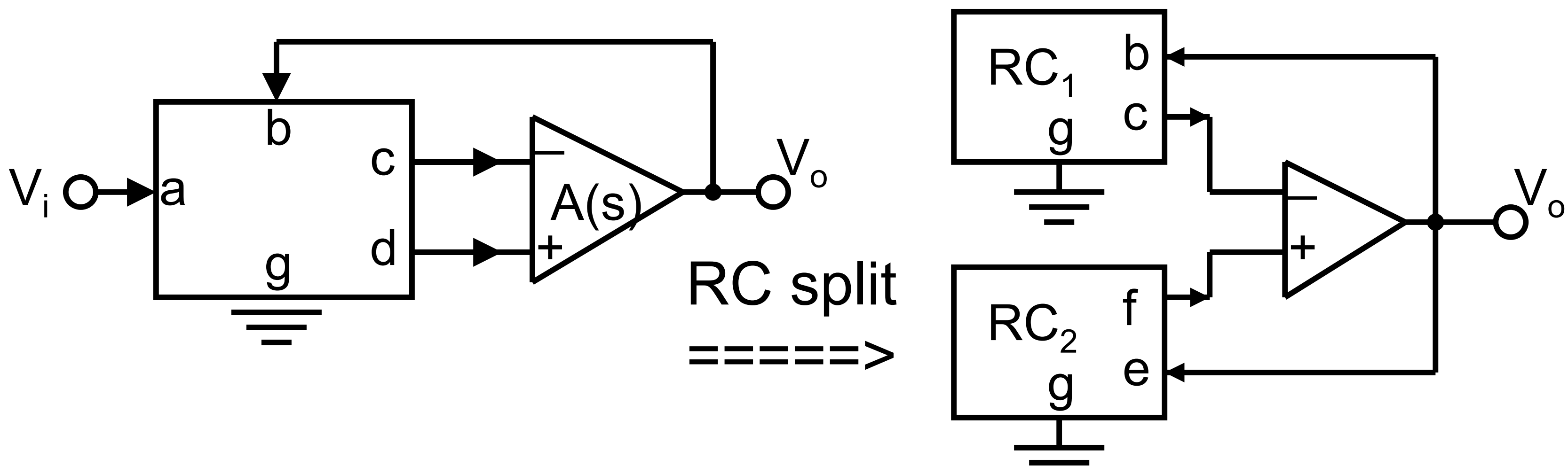


– In addition, the other element is protially lifted off ground and connected to input.



Generation of transfer function poles

- RC network is split into two parts (input node a is ignored)



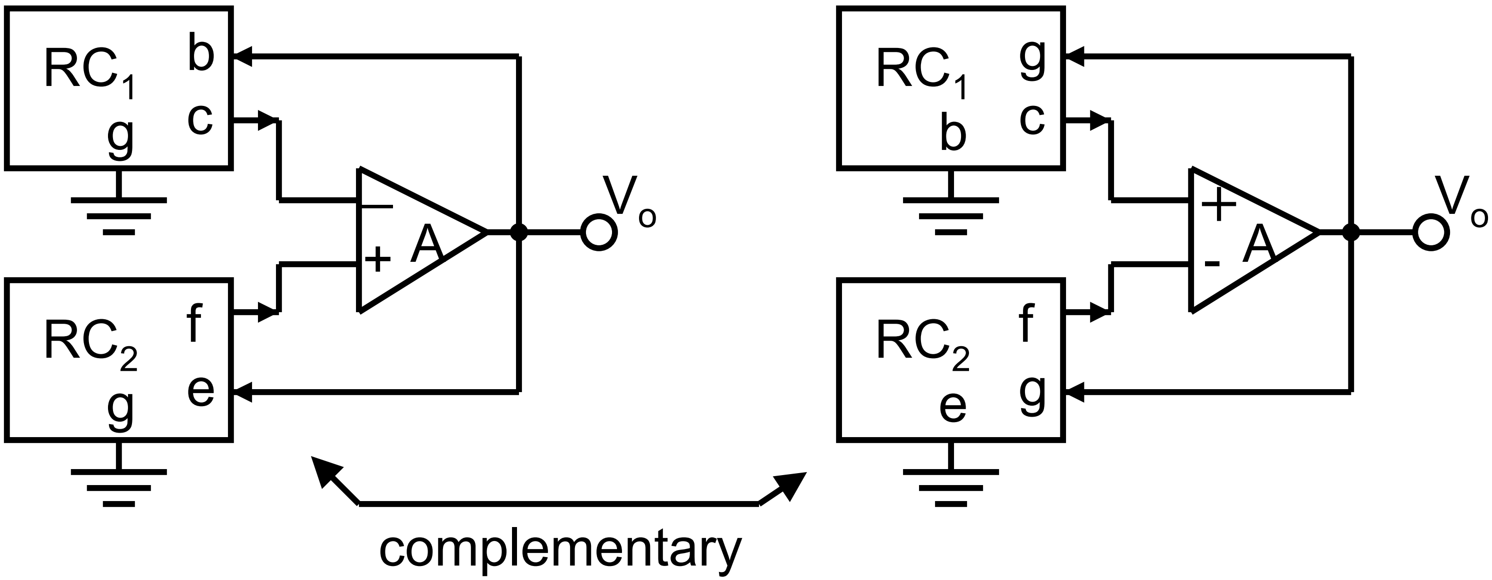
Poles determined by $T_{cb}(s)-T_{db}(s)+1/A(s)=0$

Poles determined by $T_{cb}(s)-T_{fe}(s)+1/A(s)=0$

- Complementary network
  - interchanges 1. two OPAMP input terminals each other
    2. OPAMP output with ground
  - natural frequency is the same as that of the original network

Generation of transfer function poles(cont.)

– Example(V_i ignored)



Poles ar determined by $T_{cb}(s)$-$T_{fe}(s)$+1/A(s)=0

Poles are determined by $T_{fg}(s)$-$T_{cg}(s)$+1/A(s)=0

$$\begin{cases} T_{fg}(s)=\dfrac{V_{fe}(s)}{V_{ge}(s)}=\dfrac{V_{fg}(s)+V_{ge}(s)}{V_{ge}(s)}=1+\dfrac{V_{fg}(s)}{V_{ge}(s)}=1-\dfrac{V_{fg}(s)}{V_{eg}(s)}=1-T_{fe}(s) \quad \text{-----(A)} \\ T_{cg}(s)=\dfrac{V_{cb}(s)}{V_{gb}(s)}=\dfrac{V_{cg}(s)+V_{gb}(s)}{V_{gb}(s)}=1+\dfrac{V_{cg}(s)}{V_{gb}(s)}=1-\dfrac{V_{cg}(s)}{V_{bg}(s)}=1-T_{cb}(s) \quad \text{-----(B)} \end{cases}$$

Generation of transfer function poles(cont.)

=> $T_{fg}(s)-T_{cg}(s)+1/A(s)=1-T_{fe}(s)-(1-T_{cb}(s))+1/A(s)=T_{cb}(s)-T_{fe}(s)+1/A(s)$

=> The above two systems are complementary

- From Eqs. (A) and (B)

  => Input and ground terminals in a network are complements

  => If the transfer function from input to output is H(s), then interchanging input and ground connections gives the transfer function $H_c(s)=1-H(s)$

  – Examples:

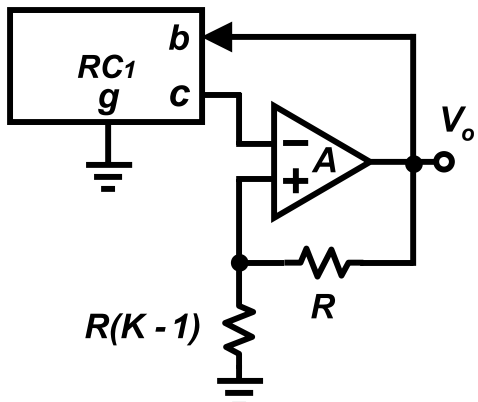
  (1) If we have a bandpass circuit realizing $H_{BP}(s)=\frac{as}{s^2+as+b}$, then interchanging input and ground results in a notch filter $H_N(s)=1-H_{BP}(s)=\frac{s^2+b}{s^2+as+b}$

  (2) If $H_{BP}(s)=\frac{2as}{s^2+as+b}$, then $1\text{-}H_{BP}(s)=H_{AP}(s)=\frac{s^2-as+b}{s^2+as+b}$

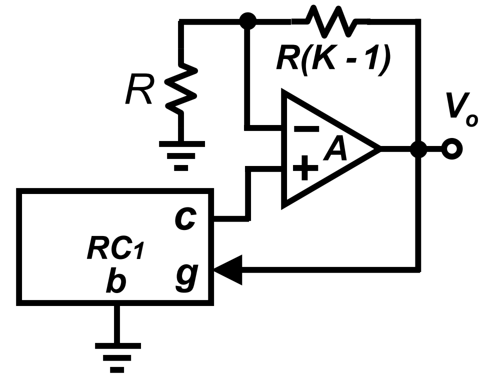
Enhanced Feedback Structure

- Enhanced negative feedback (ENF) and enhanced positive feedback (EPF) (They both are complementary each other)
  – structure (input not shown)

  1. ENF



  2. EPF



  – Positive feedback in ENF results in Q enhancement.

  Its complementary structure, EPF, also has Q enhancement.

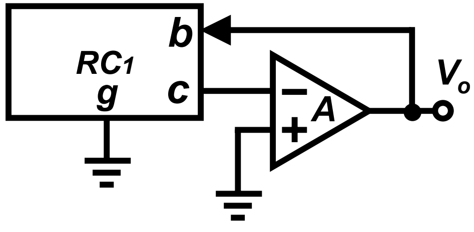
Enhanced Feedback Structure (Cont.)

– Their pole positions are given by

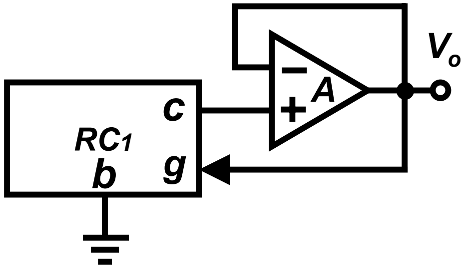
$$\underbrace{T_{cb}(s)-\frac{K-1}{K}+\frac{1}{A(s)}}_{\text{ENF}}=\underbrace{1-T_{cg}(s)-\frac{K-1}{K}+\frac{1}{A(s)}=\frac{1}{K}-T_{cg}(s)+\frac{1}{A(s)}}_{\text{EPF}}=0$$

- Special cases : Infinite-gain negative-feedback (NF) and unity-gain positive-feedback (PF)
  – structure (input not shown)

1. NF



2. PF



Enhanced Feedback Structure (Cont.)

– NF is a special case of ENF

  Its complementary structure, PF, is a special case of EPF.

– Dependence of pole positions of the ENF and the EPF networks on circuit element is identical.

– For NF&PF, pole sensitivity is the same for designing filters with either unity-gain or infinite-gain OPAMP.

– Their pole positions are given by

$$\underbrace{T_{cb}(s)+\frac{1}{A(s)}}_{\text{NF}}=\underbrace{1-T_{cg}(s)+\frac{1}{A(s)}}_{\text{PF}}=0$$

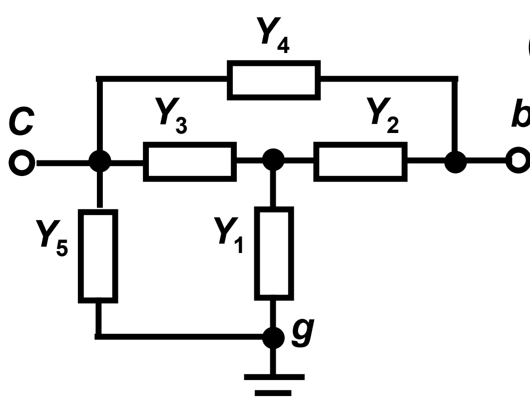
Passive RC Twoports for Single-Amplifier Biquad

- RC network of feedback filter structure, ENF or EPF, should be used to realize transfer functions $T_{cb}(s)$ or $T_{cg}(s)$ of the form of

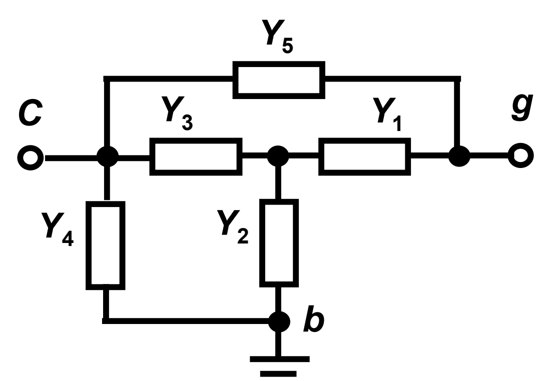
$$\mathbf{T(s)} = \frac{\mathbf{a_2 s^2 + a_1 s + a_0}}{\mathbf{s^2 + s\frac{w_1}{Q_p} + w_1^{\,2}}}$$ (for poles generation of single-amplifier biquad)

- Loaded bridged T (LBT)
  - complementary to itself
  - pole generation

LBT circuit

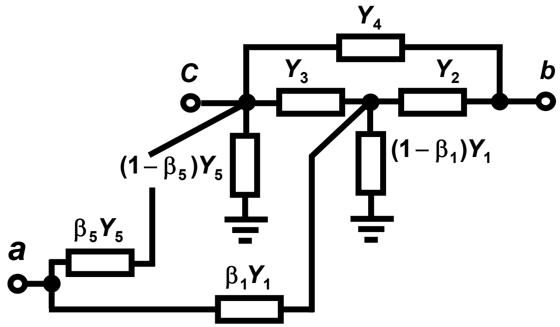


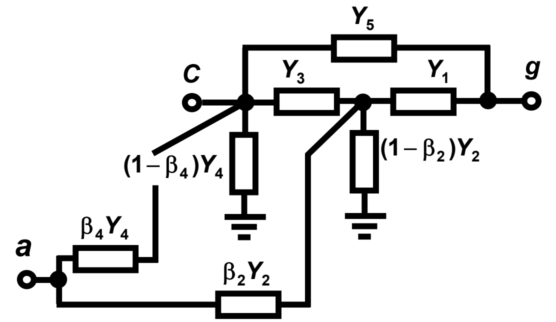
Complement of LBT



Passive RC Twoports for Single-Amplifier Biquad(Cont.)

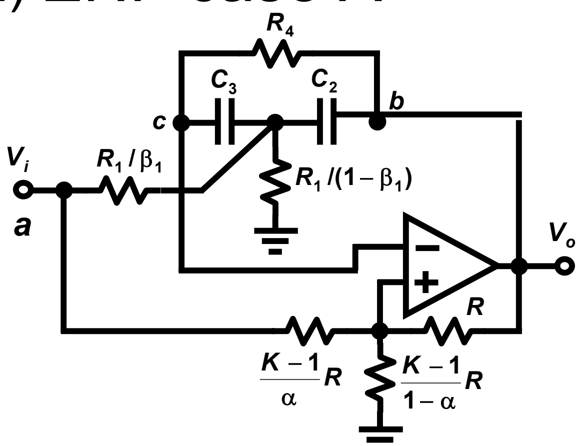
– Pole and zero generation



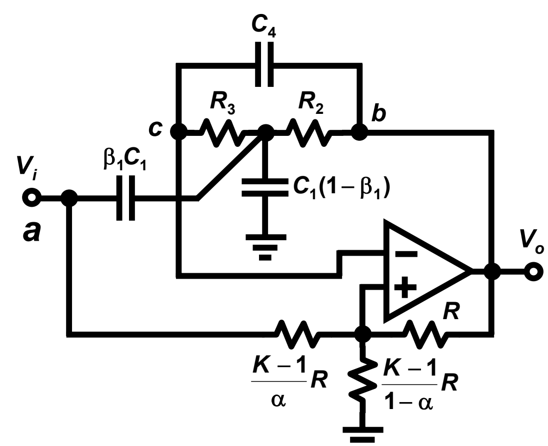


– Application example 1: $\frac{V_0}{V_1} = -\frac{\alpha}{K-1}\frac{1-\frac{K}{\alpha}T_{cg}}{1-\frac{K}{K-1}T_{cb}}$ (using RC bridged T) (suitable for ENF)

(i) ENF case A



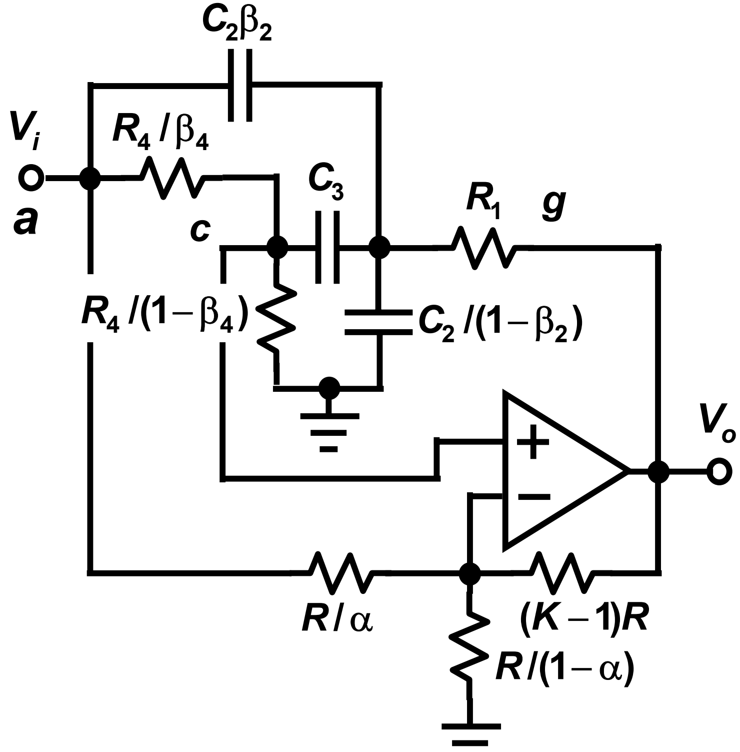
(ii) ENF case B



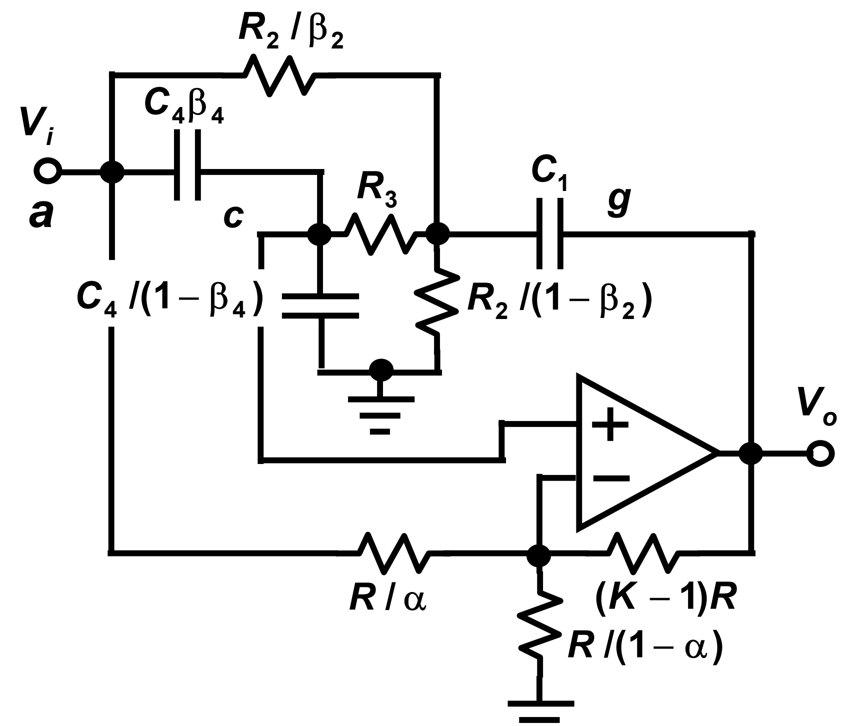
Passive RC Twoports for Single-Amplifier Biquad(Cont.)

– Application example 2: (suitable for EPF) $\frac{V_0}{V_1} = -K \frac{\alpha \frac{K-1}{K} - T_{cg}}{1 - KT_{cg}}$ (using RC ladder, complementary to RC bridged T)

(i) EPF case A

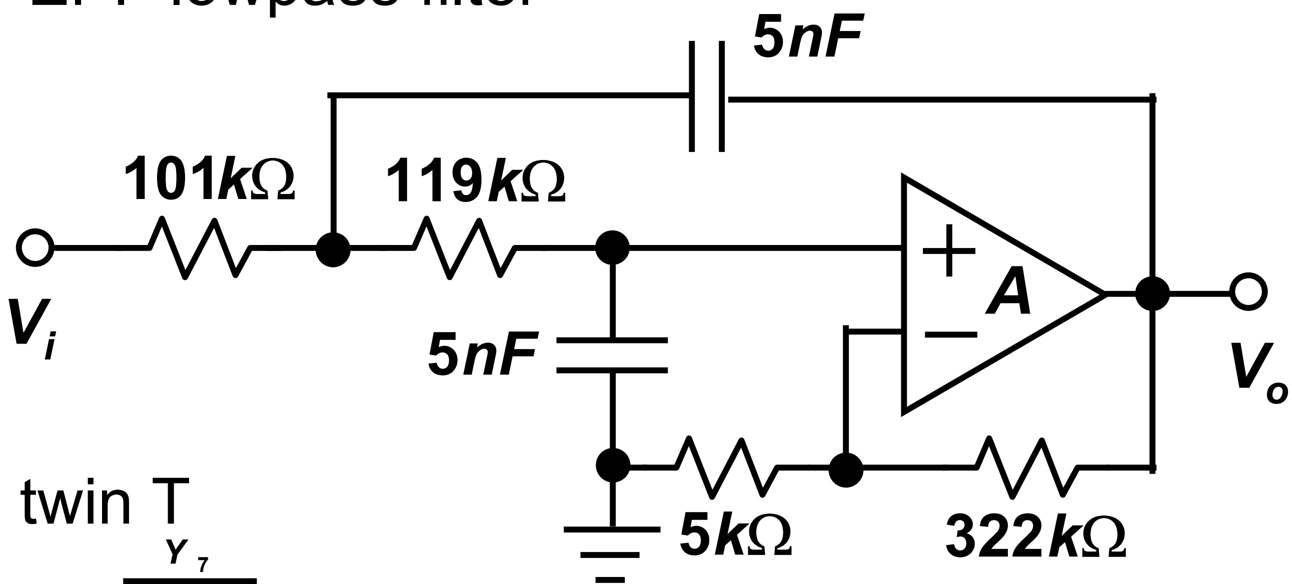


(ii) EPF case B

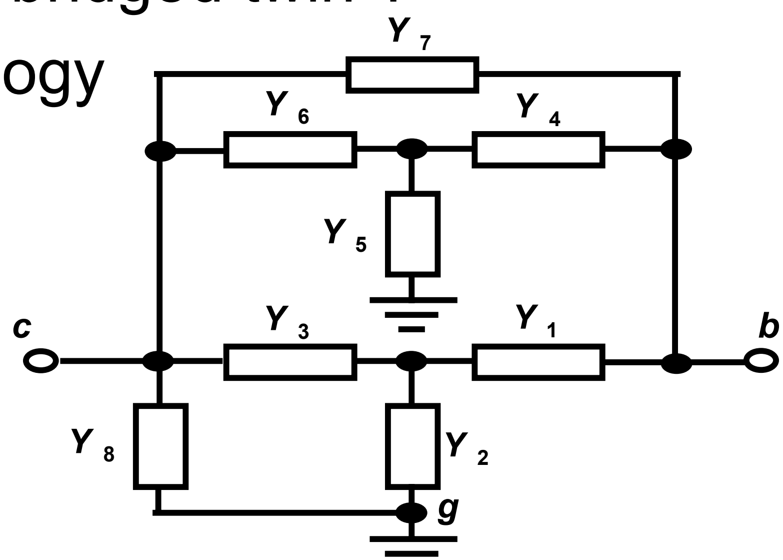


Passive RC Twoports for Single-Amplifier Biquad (Cont.)

– Sallen & Key EPF lowpass filter

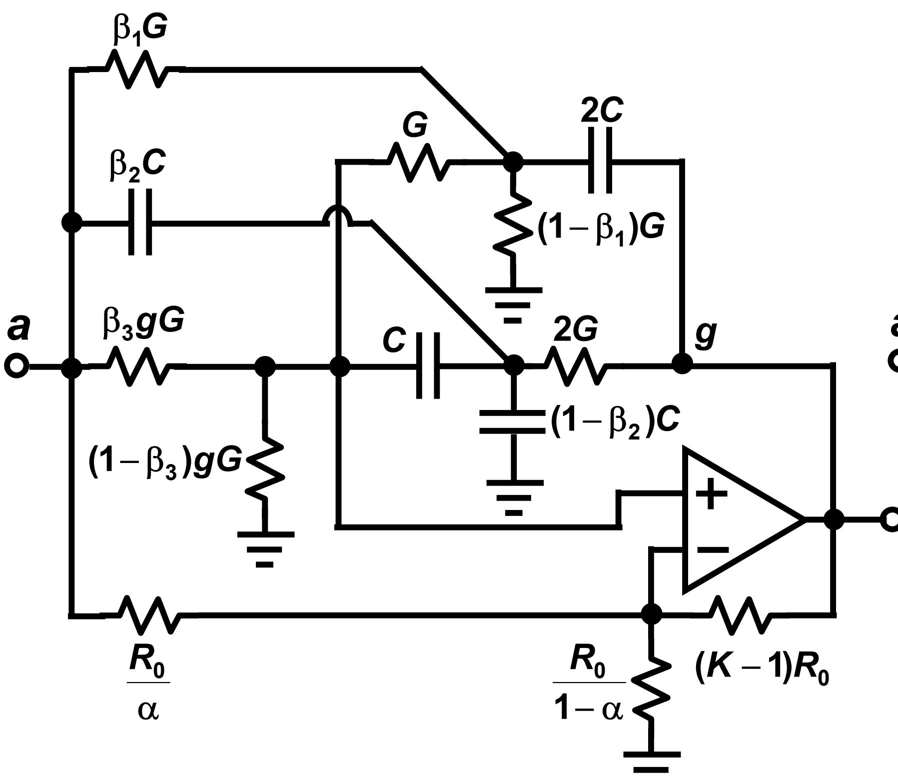


- Loaded bridged twin T
  - topology

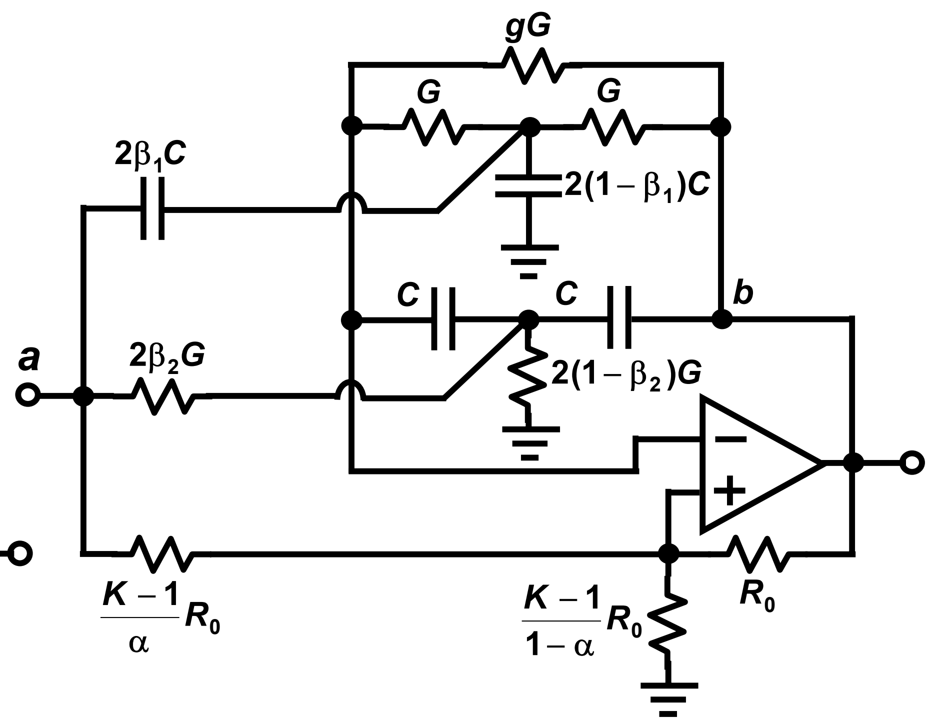


Passive RC Twoports for Single-Amplifier Biquad(Cont.)

– ENF example



– EPF example



Multiamplifier Biquads

- Biquads using a composite amplifier

  $\frac{\Delta w_0}{w_0}$ and $\frac{\Delta Q_0}{Q_0}$ are proportional to $\left|\frac{1}{A(jw_0)}\right|^2$, whereas for the SAB they were proportional to $\left|\frac{1}{A(jw_0)}\right|$.

- Biquads based on general impedance converter (GIC)

  Biquads based on Antoniou's general impedance converter has the best sensitivity performance among the many proposed two-amplifier circuits.

Multiamplifier Biquads

- Biquads using three OPAMPs
  - state-variable realization first proposed by Kerwin, Huelsman, and Newcomb (KHN)
  - greater versatility

    (1) different types of transfer functions to be realized simultaneously at the available OPAMP outputs

    (2) tuning will be noninteractive because the additional OPAMPs isolate different parts of the circuit from each other.

Multiamplifier Biquads VS. SABs

- Single-amplifier biquads (SABs)

  2nd-order RC network together with a single amplifier can be used to realize an arbitrary 2nd-order transfer function
- Multiamplifier circuits
  - Advantages

    (1) simultaneous availability of different transfer functions at different outputs

    (2) the ease of tuning and noninteractive adjusting of filter parameters, because the design equations are seen to be decoupled by the insertion of additional OPAMPs

Multiamplifier Biquads VS. SABs (Cont.)

- Disadvantage

  increased expense of additional OPAMPs and power consumption
- Sensitivity is not significantly better than well-designed SABs

- If low sensitivity, in particular to the active device tolerances, is of major concern, composite amplifier may be preferable. SABs based on composite amplifier has small w_0 and Q errors which is ideally proportional to $\left|\frac{1}{A(jw_0)}\right|^2$ instead of $\left|\frac{1}{A(jw_0)}\right|$ of the general one OPAMP approach