Laboratory #7
Filters

I. Objectives

1. Understand the basic properties of filters.
2. Familiarize with some simple single-OPAMP filters

II. Components and Instruments

1. Components
   (1) LM324 ×1
   (2) Resistor: 3.3kΩ ×2, 10kΩ ×2, 100kΩ ×4, 1MΩ ×1
   (3) Capacitor: 0.01μF ×3, 0.1μF ×1, 10μF ×1, 100μF ×1
2. Instruments
   (1) DC power supply (Keysight E36311A)
   (2) Oscilloscope (Agilent MSOX 2014A)

III. Reading

1. Section 11-1 to 11-4 of "Microelectronics Circuits 6th edition, Sedra/Smith".
2. Experiment 11 of "Laboratory Explorations for Microelectronic Circuits 4th edition, Kenneth C. Smith".

IV. Preparation

1. Introduction

In this Lab, we will learn the properties of an important block in communication and instrumentation systems, the electronic filter. The filters perform a frequency-selection function: passing signals whose frequency spectrum lies within a specified range, and stopping signals whose frequency spectrum falls outside this range. Such a filter has ideally a frequency band (or bands) over which the magnitude of transmission is unity (the filter passband) and a frequency band (or bands) over which the transmission is zero (the filter stopband). Fig. 7.1 depicts the ideal transmission characteristics of the four major filters types: low-pass (LP) in Fig. 7.1(a), high-pass (HP) in Fig. 7.1(b), band-pass (BP) in Fig. 7.1(c), and band-stop (BS) in Fig. 7.1(d). These idealized characteristics, by virtue of their vertical edges, are known as brick-wall response.

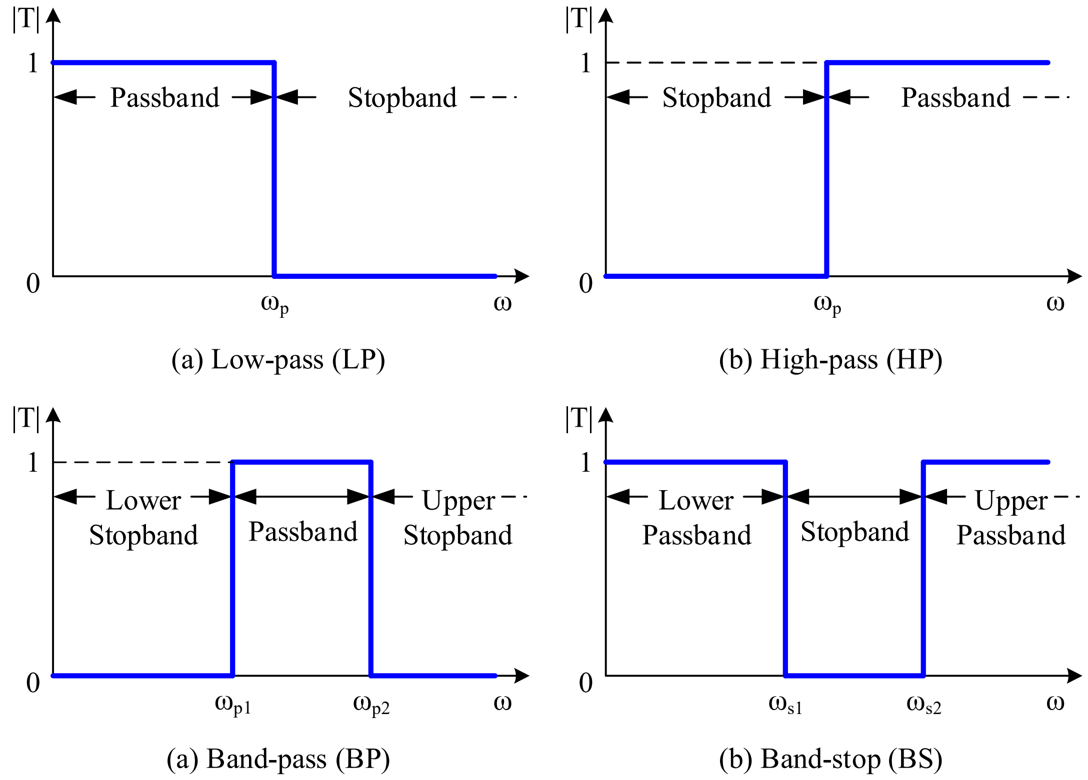


Fig. 7.1 Ideal transmission characteristics of the four types of filter:
(a) Low-pass (LP), (b) High-pass (HP), (c) Band-pass (BP), (d) Band-stop (BS)

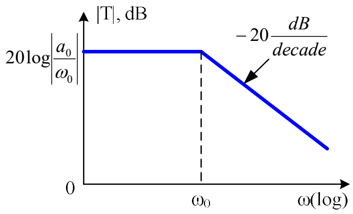
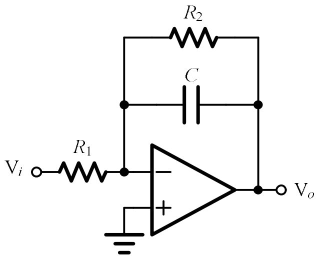
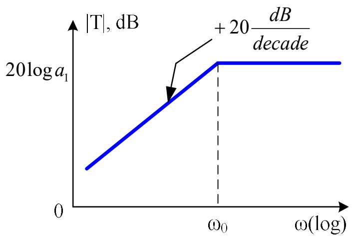
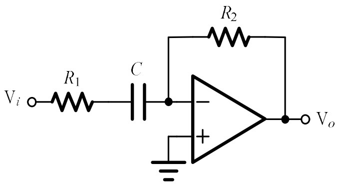
2. First-order LP and HP filters

The general first-order transfer function is given by

$$T(s) = \frac{a_1 s + a_0}{s + \omega_0} \quad \text{........(Eq. 7.1)}$$

This bilinear transfer function characterizes a first-order filter with a natural mode at $s = -\omega_0$, a transmission zero at $s = -a_0/a_1$, and a high-frequency gain that approaches a_1. The numerator coefficients (a_0 and a_1) determine the types of filters (e.g., low pass, high pass, etc.). The realizations of active (OPAMP-RC) low-pass and high-pass filters are shown in Table 7.1.

Table 7.1 Active first-order filter

Filter Type and T(s)	Bode Plot for |T|	OPAMP-RC Realization
(a)Low pass (LP) $T(s)=\dfrac{a_0}{s+\omega_0}$		 $CR_2=\dfrac{1}{\omega_0}$, $DC\,gain=\dfrac{R_2}{R_1}$
(b)High pass (HP) $T(s)=\dfrac{a_1 s}{s+\omega_0}$		 $CR_1=\dfrac{1}{\omega_0}$, $High\text{-}frequency\,gain=\dfrac{R_2}{R_1}$

3. Second-order LP and HP filters

In many signal processing and communication applications, a high selectivity (sharp transition) is required because the interferer frequency is close to the desired signal band. High selectivity can be simply achieve by increasing the "order" of the transfer function, shown as Fig. 7.2. Assume a first-order low-pass filter has single pole which attenuates at the rate of -20dB/decade. By changing the first-order to the second-order, the slope of attenuation will be sharpen in a result.

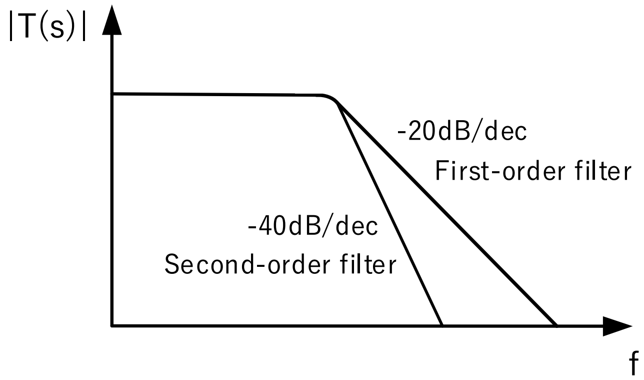


Fig. 7.2 Frequency response of first-order and second-order filter

The general second-order (or biquadratic) filter transfer function is usually expressed in the standard form

$$T(s)=\frac{a_2 s^2+a_1 s+a_0}{s^2+(\omega_0/Q)s+\omega_0} \quad \text{........(Eq. 7.2)}$$

where ω_0 and Q determine the natural modes (poles) according to

$$p_1, p_2=-\frac{\omega_0}{2Q}\pm j\omega_0\sqrt{1-\left(1/4Q^2\right)} \quad \text{........(Eq. 7.3)}$$

As shown in Fig. 7.3, the parameter ω_0 (pole frequency) determines the radial distance (r) from the origin, and the parameter Q (pole quality factor) determines the distance (d) of the poles from the imaginary axis. The effects of different Q value to a system shown below:

1. Q is negative, poles are in the right plane which makes system be unstable and produces oscillation.
2. Q is positive and lower than $1/\sqrt{2}$, system will not oscillate and no peaking in the step response. However, the response of system is slower.
3. Q equals to $1/\sqrt{2}$, system will have widest passband without any peaking in the frequency response.
4. Q is greater than $1/\sqrt{2}$, system will have overshoot in the step response and ringing before converging to a steady-state value.
5. Q is infinite, poles are located at imaginary axis which causes sustained oscillations in circuit realization.

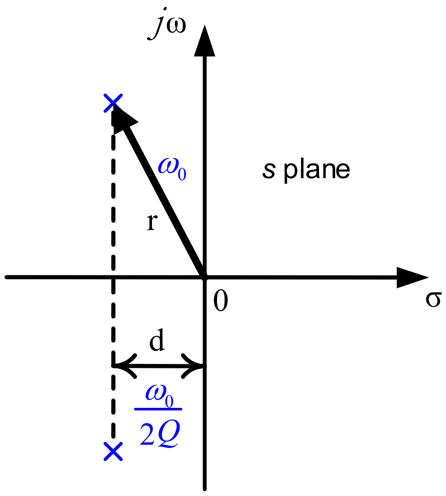
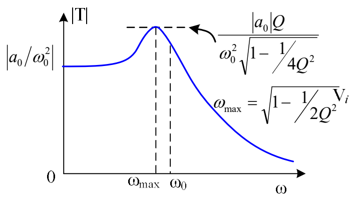
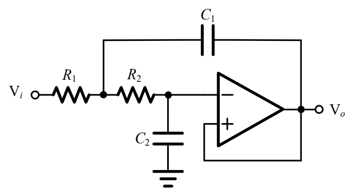
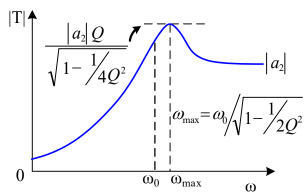
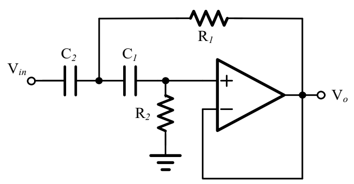


Fig. 7.3 Location of the pair of complex-conjugate poles in the s plane.

Sallen-and-Key filter is one of the most widely-used topology to implement second-order filter. The advantage of this configuration is that it only utilizes an amplifier, two resistors and two capacitors to realize low-pass, high-pass or band-pass filters without using inductors. Inductor-less design provides lower noise effect to circuit due to low electromagnetic interference (EMI), and decreases circuit-design complexity. And, the circuit implementations and bode plots of second-order filters are illustrated in Table 7.2.

Table 7.2 Active second-order filter

Filter Type and T(s)	Bode Plot for |T|	OPAMP-RC Realization
(a)Low pass (LP) $T(s)=\frac{a_0}{s^2+(\omega_0/Q)s+\omega_0}$ $DC\ gain=\frac{a_0}{\omega_0^2}$		
(b)High pass (HP) $T(s)=\frac{a_2 s^2}{s^2+(\omega_o/Q)s+\omega_o}$		

V. Exploration

1. Measure the frequency response of first-order low-pass filter

(1) Complete the wiring as the figure below.

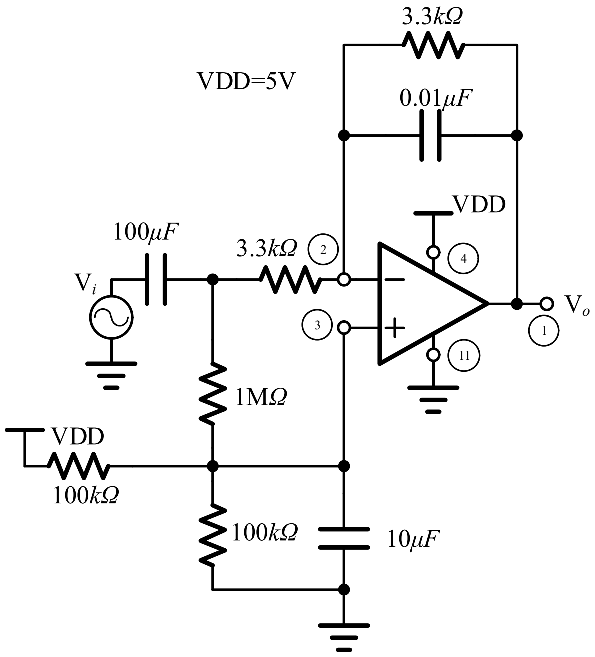


Fig. 7.4 First-order low-pass filter

(2) Varies the frequency of input signal (function generator, SINE wave), record the peak-to-peak values of the output voltage in the Table 7.3.

Table 7.3

Freq. (Hz)	$V_{i,p\text{-}p}$ (mV)	$V_{o,p\text{-}p}$ (mV)	$V_{o,p\text{-}p}$ / $V_{i,p\text{-}p}$ (dB)
20	200		
100	200		
1k	200		
5k	200		
10k	200		
12k	200		
14k	200		
16k	200		
18k	200		
20k	200		
50k	200		
100k	200		

(3) Draw the figure of “($V_{o,p\text{-}p}$/$V_{i,p\text{-}p}$) to freq.”.

2. Measure the frequency response of the first-order high-pass filter

(1) Complete the wiring as the figure below.

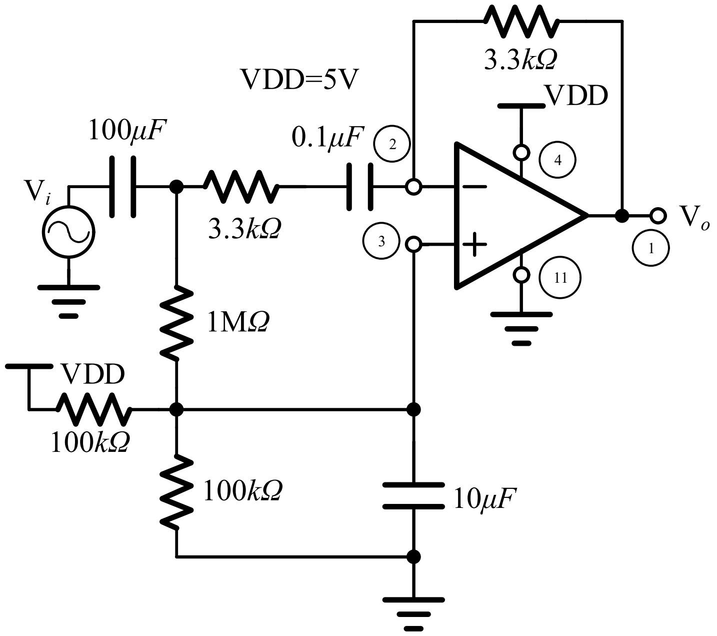


Fig. 7.5 First-order high-pass filter

(2) Varies the frequency of input signal (function generator, SINE wave), record the peak-to-peak values of the output voltage in the Table 7.4.

Table 7.4

Freq. (Hz)	$V_{i,p\text{-}p}$ (mV)	$V_{o,p\text{-}p}$ (mV)	$V_{o,p\text{-}p}$ / $V_{i,p\text{-}p}$ (dB)
20	200		
100	200		
1k	200		
1.5k	200		
2k	200		
2.5k	200		
5k	200		
10k	200		
20k	200		
50k	200		
100k	200		
200k	200		

(3) Draw the figure of “($V_{o,p\text{-}p}$/$V_{i,p\text{-}p}$) to freq.”.

3. Measure the frequency response of the second-order low-pass filter

(1) Complete the wiring as the figure below.

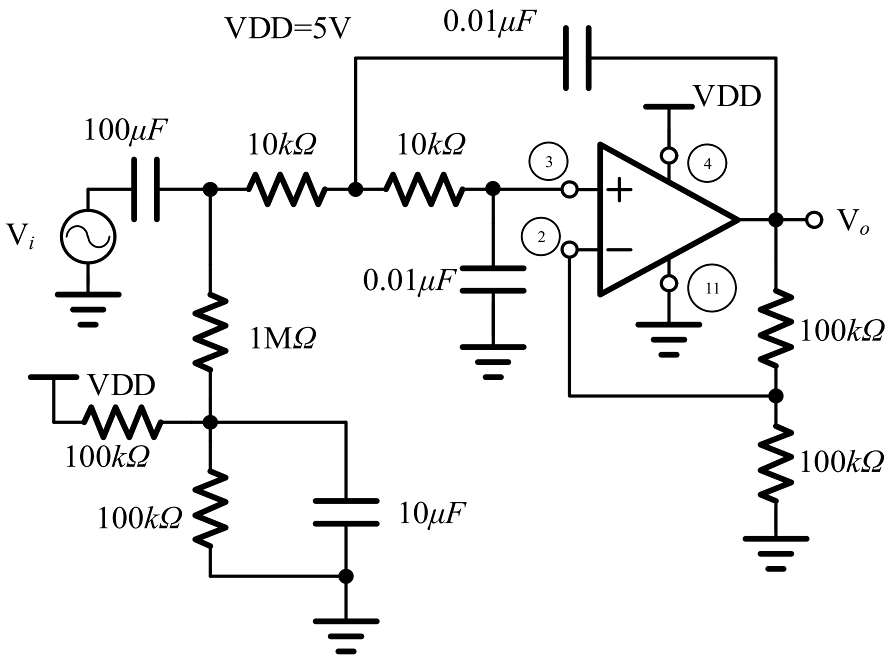


Fig. 7.6 Second-order low-pass filter

(2) Varies the frequency of input signal (function generator, SINE wave), record the peak-to-peak values of the output voltage in the Table 7.5.

Table 7.5

Freq. (Hz)	$V_{i,p\text{-}p}$ (mV)	$V_{o,p\text{-}p}$ (mV)	$V_{o,p\text{-}p}$ / $V_{i,p\text{-}p}$ (dB)
20	200		
50	200		
100	200		
1k	200		
1.5k	200		
2k	200		
2.25k	200		
2.5k	200		
2.75k	200		
3k	200		
10k	200		
100k	200		

(3) Draw the figure of "($V_{o,p\text{-}p}$/$V_{i,p\text{-}p}$) to freq.".

4. Measure the frequency response of the second-order high-pass filter **(Optional)**

(1) Complete the wiring as the figure below.

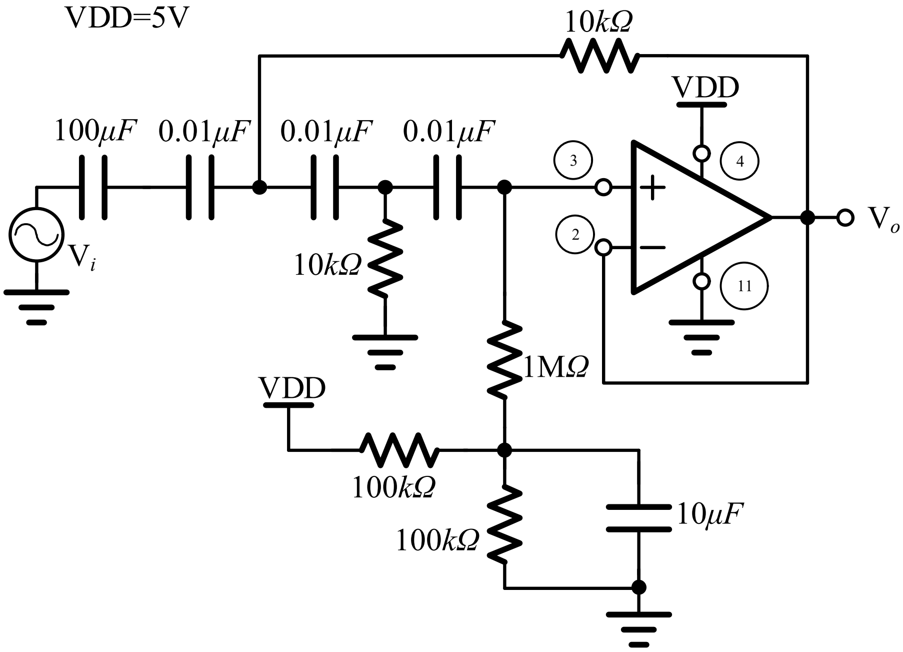


Fig. 7.7 Second-order high-pass filter

(2) Varies the frequency of input signal (function generator, SINE wave), record the peak-to-peak values of the output voltage in the Table 7.6.

Table 7.6

Freq. (Hz)	$V_{i,p\text{-}p}$ (mV)	$V_{o,p\text{-}p}$ (mV)	$V_{o,p\text{-}p}$ / $V_{i,p\text{-}p}$ (dB)
20	200		
50	200		
100	200		
1k	200		
1.5k	200		
2k	200		
2.25k	200		
2.5k	200		
2.75k	200		
3k	200		
10k	200		
100k	200		

(3) Draw the figure of "($V_{o,p\text{-}p}$/$V_{i,p\text{-}p}$) to freq.".

Laboratory #7 Pre-lab

Class:

Name: Student ID:

1. Explore the frequency response of the first-order low-pass filter,
   (1) Use PSpice to do the ac analysis on the circuit below, and show the plot of frequency response of Vo/Vi (dB)

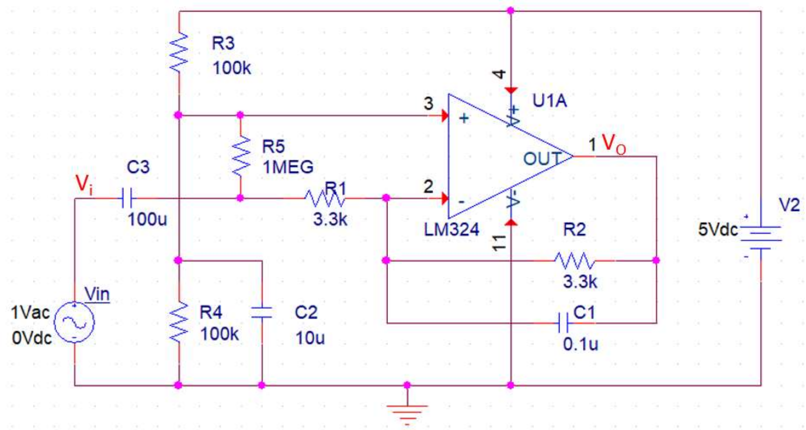


Fig. 7.8 Schematic of the first-order low-pass filter for AC analysis

   (2) Change C1 from 0.1μF to 1μF and 10μF respectively, and show the plot of frequency response of Vo/Vi (dB).
   (3) Explain the relationship between the capacitor C1 and the 3-dB frequency.

2. Explore the frequency response of first-order high-pass filter,

(1) Use PSpice to do the ac analysis on the circuit below, and show the plot of frequency response of Vo/Vi (dB)

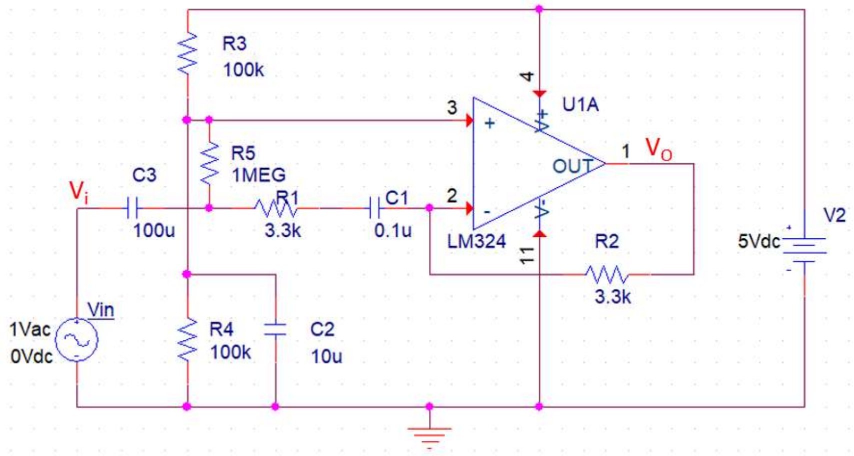


Fig. 7.9 Schematic of the first-order high-pass filter for AC analysis

(2) Change C1 from 0.1μ to 1μ and 10μ respectively, and show the plot of frequency response of Vo/Vi (dB)

(3) Explain the relationship between the capacitor C1 and the 3-dB frequency.

3. Explore the frequency response of second-order low-pass filter,

(1) Use PSpice to do the ac analysis on the circuit below, and show the plot of frequency response of Vo/Vi (dB).

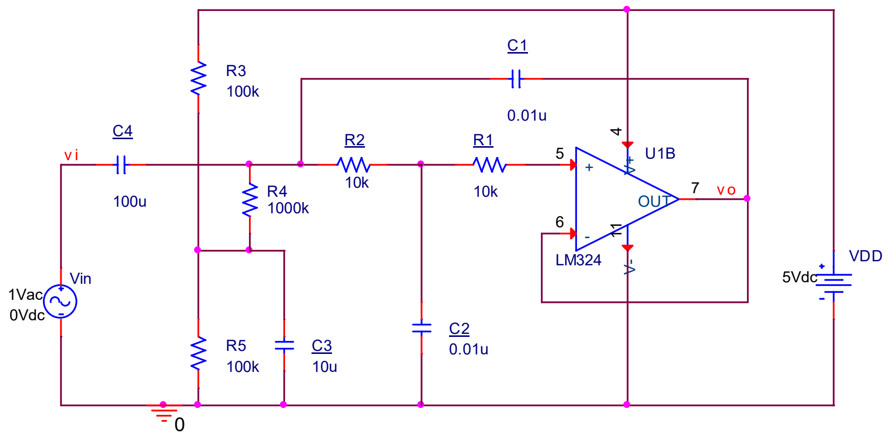


Fig. 7.10 Schematic of the second-order low-pass filter for AC analysis

(2) Change C1 and C2 from 0.01μ to 1μ and 10μ respectively, and show the plot of frequency response of Vo/Vi (dB)

(3) Explain the relationship between the capacitor (C1 and C2) and the 3-dB frequency.

4. Explore the frequency response of second-order high-pass filter **(Optional)**

(1) Use PSpice to do the ac analysis on the circuit below, and show the plot of frequency response of Vo/Vi (dB).

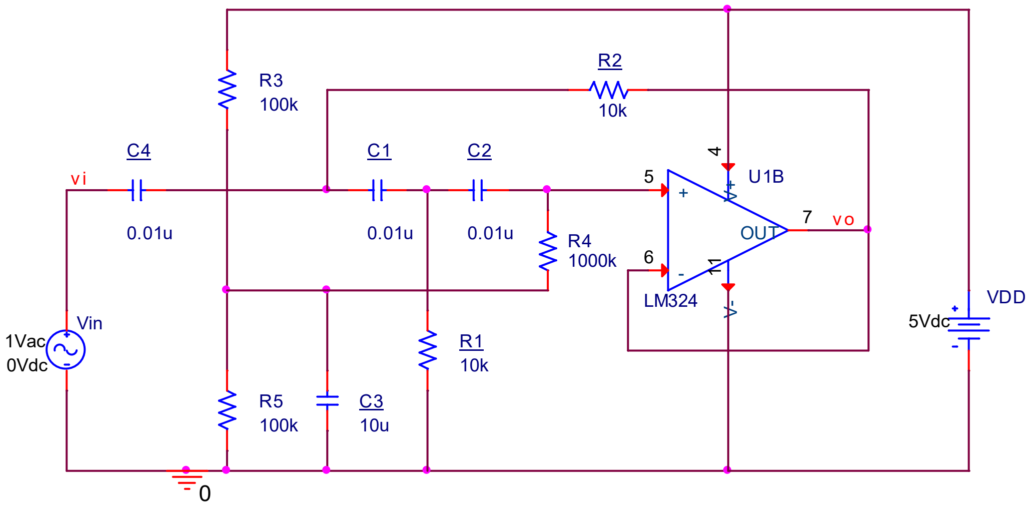


Fig. 7.11 Schematic of the second-order high-pass filter for AC analysis

(2) Change C1 and C2 from 0.01μ to 1μ and 10μ respectively, and show the plot of frequency response of Vo/Vi (dB)

(3) Explain the relationship between the capacitor (C1 and C2) and the 3-dB frequency.

Laboratory #7 Report

Class:

Name: Student ID:

1. Exploration 1

(1) Experimental result

Table 7.3

Freq. (Hz)	$V_{i,p\text{-}p}$ (mV)	$V_{o,p\text{-}p}$ (mV)	$V_{o,p\text{-}p}$ / $V_{i,p\text{-}p}$ (dB)
20	200		
100	200		
1k	200		
5k	200		
10k	200		
12k	200		
14k	200		
16k	200		
18k	200		
20k	200		
50k	200		
100k	200		

(2) The figure of "($V_{o,p\text{-}p}/V_{i,p\text{-}p}$) to freq" f_{3dB} = _____Hz

2. Exploration 2

(1) Experimental result

Table 7.4

Freq. (Hz)	$V_{i,p\text{-}p}$ (mV)	$V_{o,p\text{-}p}$ (mV)	$V_{o,p\text{-}p}$ / $V_{i,p\text{-}p}$ (dB)
20	200		
100	200		
1k	200		
1.5k	200		
2k	200		
2.5k	200		
5k	200		
10k	200		
20k	200		
50k	200		
100k	200		
200k	200		

(2) The figure of "($V_{o,p\text{-}p}/V_{i,p\text{-}p}$) to freq" f_{3dB} = _____Hz

3. Exploration 3

(1) Experimental result

Table 7.5

Freq. (Hz)	$V_{i,p-p}$ (mV)	$V_{o,p-p}$ (mV)	$V_{o,p-p}$ / $V_{i,p-p}$ (dB)
20	200		
50	200		
100	200		
1k	200		
1.5k	200		
2k	200		
2.25k	200		
2.5k	200		
2.75k	200		
3k	200		
10k	200		
100k	200		

(2) The figure of "($V_{o,p-p}$/$V_{i,p-p}$) to freq." f_{3dB} = _____Hz

4. Exploration 4 **(Optional)**

(1) Experimental result

Table 7.6

Freq. (Hz)	$V_{i,p-p}$ (mV)	$V_{o,p-p}$ (mV)	$V_{o,p-p}$ / $V_{i,p-p}$ (dB)
20	200		
50	200		
100	200		
1k	200		
1.5k	200		
2k	200		
2.25k	200		
2.5k	200		
2.75k	200		
3k	200		
10k	200		
100k	200		

(2) The figure of "($V_{o,p-p}$/$V_{i,p-p}$) to freq." f_{3dB} = _____Hz

5. Problem 1

Please derive the transfer function of second-order low-pass filter as shown below

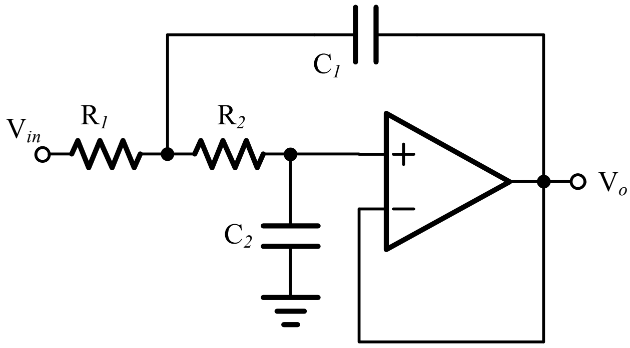


Fig. 7.12 Second-order low-pass filter

6. Problem 2

Please derive the transfer function of second-order high-pass filter as shown below

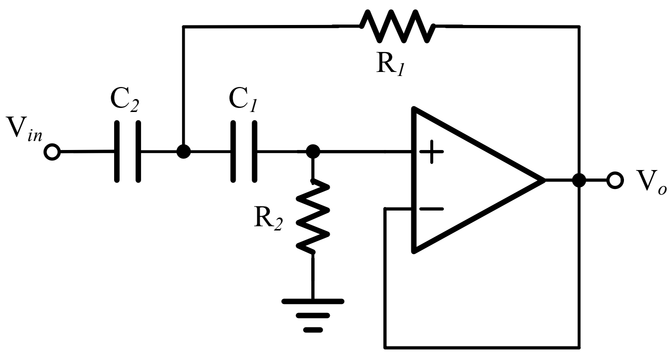


Fig. 7.13 Second-order high-pass filter

7. Conclusion