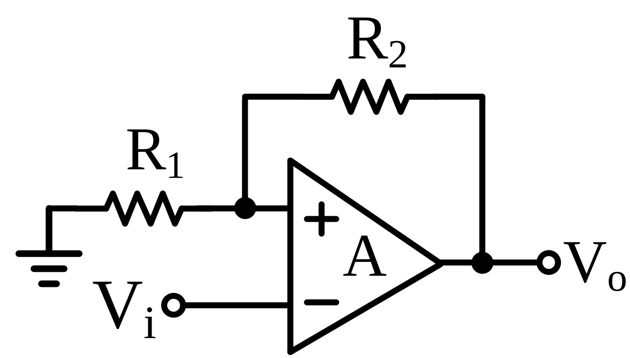
Feedback Amplifiers

- Feedback : a portion of the output is returned to the input to form part of the system excitation.
- Feedback was used to make the operating point of a transistor insensitive to both manufacturing variations and environmental changes (e.g. temperature and humidity)
  - Example
    - Feedback factor $\beta = \dfrac{R_1}{R_1 + R_2}$
    - Open-loop gain A
    - Close-loop gain A_f
    - Derivation

$$A_f = \frac{V_o}{V_i} = \frac{A}{1+\beta A}$$

When βA >> 1

$$A_f \approx \frac{1}{\beta} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$$



Feedback Amplifiers (Cont.)

- Advantages
  - Gain desensitivity
    Gain is less sensitive to component variations.
  - Smaller nonlinear distortion
  - Noise reduction/shaping
  - Increased (decreased) input resistance for voltage (current) input
    Increased (decreased) output resistance for current (voltage) output
  - Wider bandwidth
- Disadvantages
  - Reduced gain
  - Negative feedback → frequency ↗ → phase shift ↗
    Positive feedback → may oscillate (depends on phase/gain margin)
- Classification of amplifiers (Four basic amplifier topologies)
  - Voltage amplifier
  - Current amplifier
  - Transconductance amplifier
  - Transimpedance amplifier

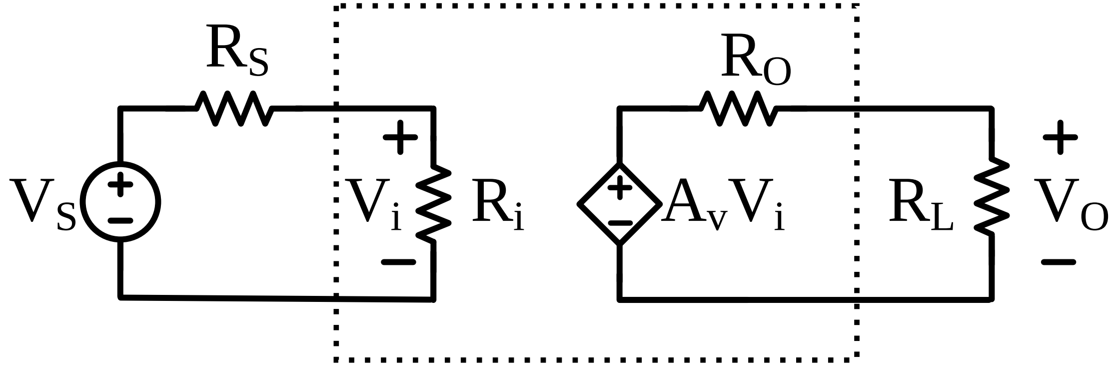
Classification of Amplifiers

- Voltage amplifier

  Voltage-controlled voltage-source

  - Equivalent circuit

$$A_v = \frac{V_o}{V_i}$$

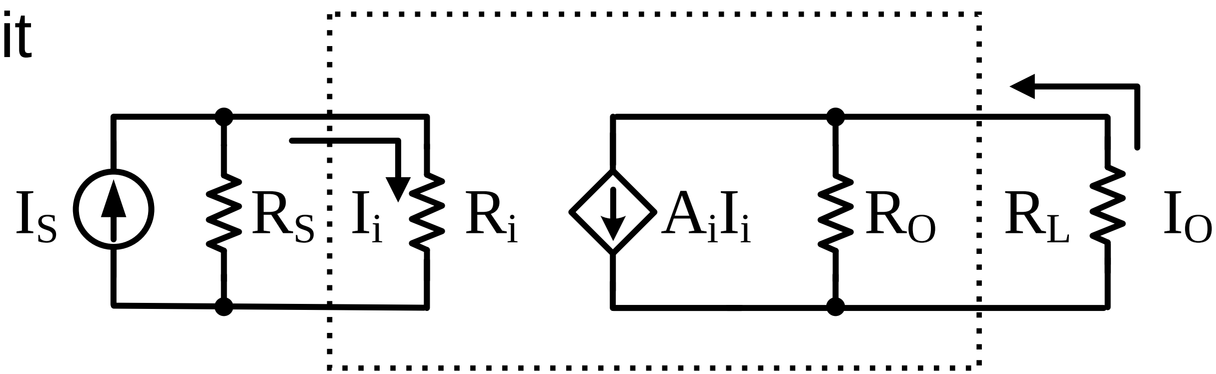


- Current amplifier

  Current-controlled current-source

  - Equivalent circuit

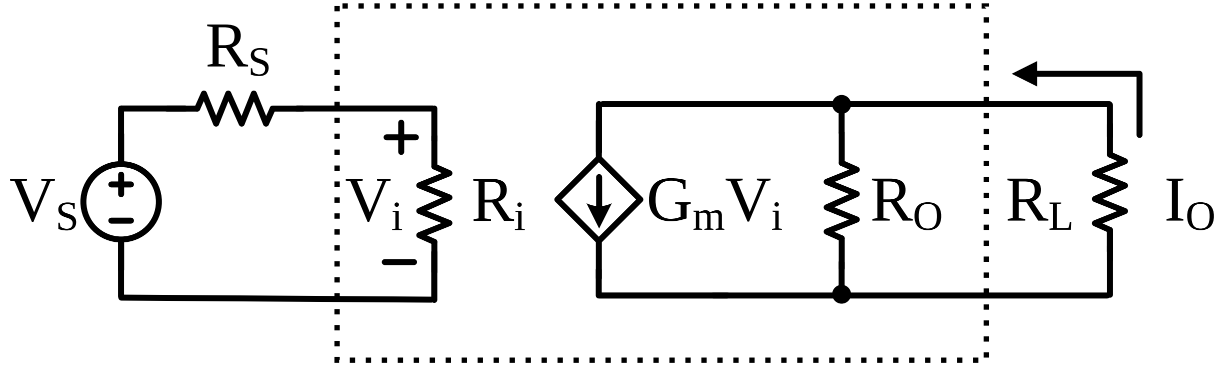
$$A_i = \frac{I_o}{I_i}$$



Classification of Amplifiers (Cont.)

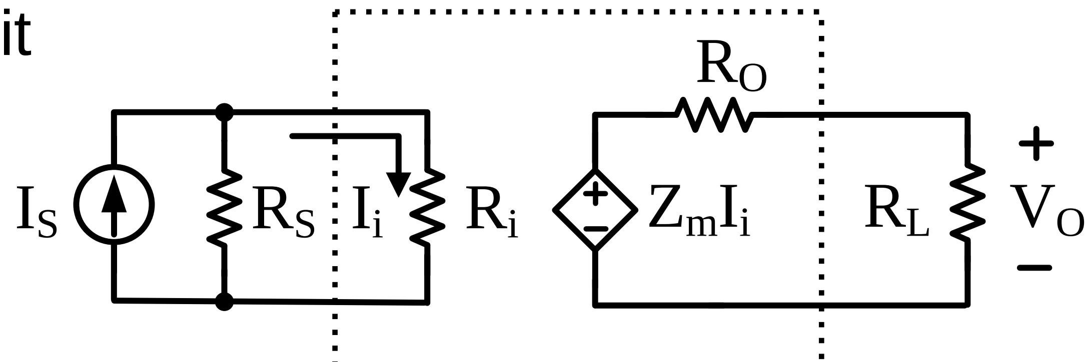
- Transconductance Amplifier (or voltage-current converter)
  Voltage-controlled current-source
  - Equivalent circuit

$$G_m = \frac{I_o}{V_i}$$



- Transimpedance Amplifier (or current-voltage converter)
  Current-controlled voltage-source
  - Equivalent circuit

$$Z_m = \frac{V_o}{I_i}$$



Classification of Amplifiers (Cont.)

- Basic Amplifier Characteristics

	Amplifier type			
	Voltage		Current	
Parameter	Ideal	Practical	Ideal	Practical
Z_I	∞	High; $\lvert Z_i \rvert >> R_s$	0	Low; $\lvert Z_i \rvert << R_s$
Z_O	0	Low; $\lvert Z_o \rvert << R_L$	∞	High; $\lvert Z_o \rvert >> R_L$
Gain or transfer ratio	$V_o = A_v V_s$	$V_o = A_v V_i \approx A_v V_s$	$I_o = A_i I_s$	$I_o = A_i I_i \approx A_i I_s$

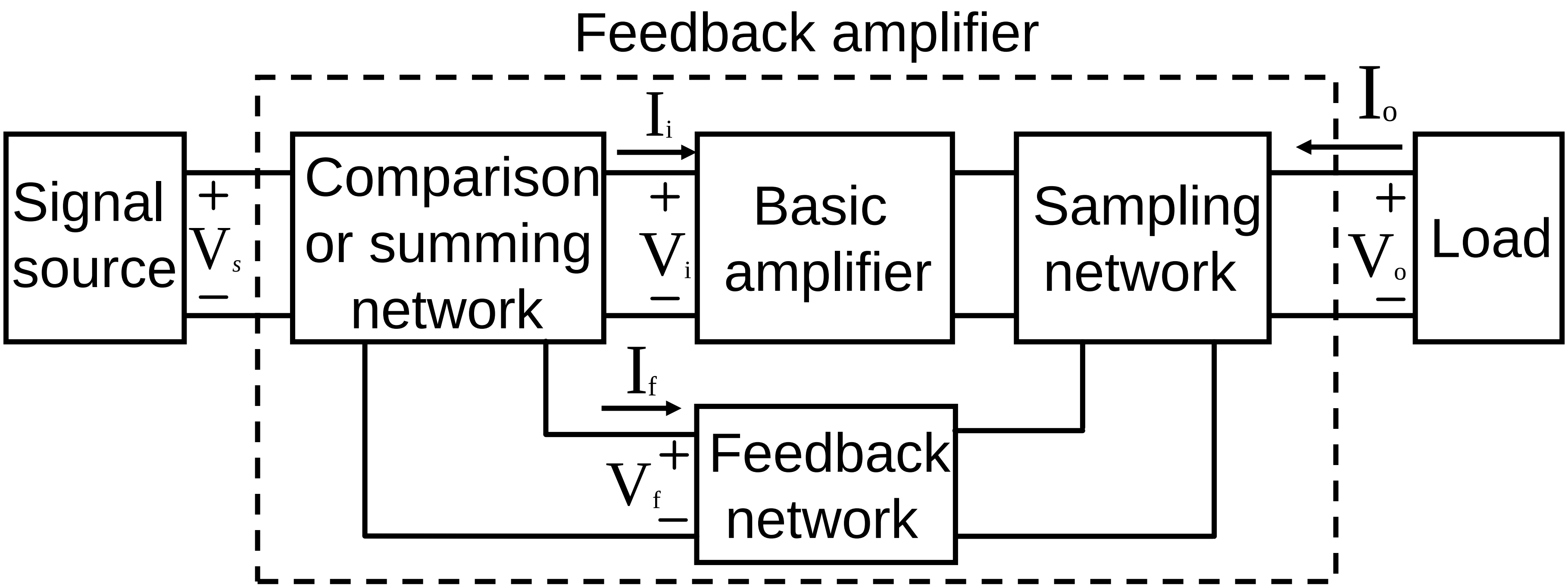
Classification of Amplifiers (Cont.)

- Basic Amplifier Characteristics

	Amplifier type			
	Transconductance		Transimpedance	
Parameter	Ideal	Practical	Ideal	Practical
Z_I	∞	High; $\lvert Z_i \rvert >> R_s$	0	Low; $\lvert Z_i \rvert << R_s$
Z_O	∞	High; $\lvert Z_o \rvert >> R_L$	0	Low; $\lvert Z_o \rvert << R_L$
Gain or transfer ratio	$I_o = G_m V_s$	$I_o = G_m V_i \approx G_m V_s$	$V_o = Z_m I_s$	$V_o = Z_m I_s \approx Z_m I_s$

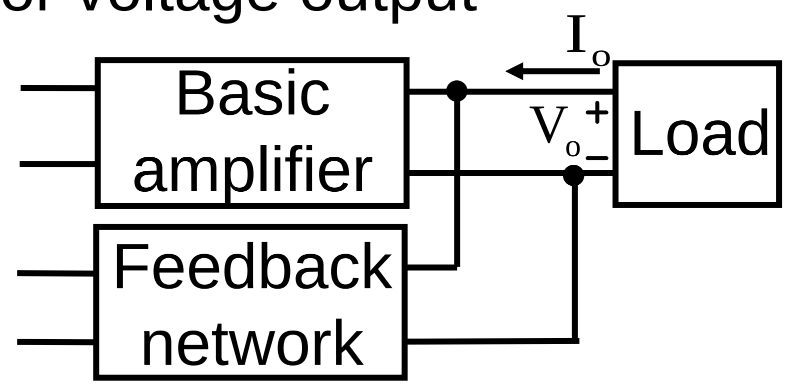
Single-Loop Feedback Amplifiers

- Basic structure
  - 4 basic building blocks
    1. sampling network
    2. comparison or summing network
    3. feedback network
    4. basic amplifier



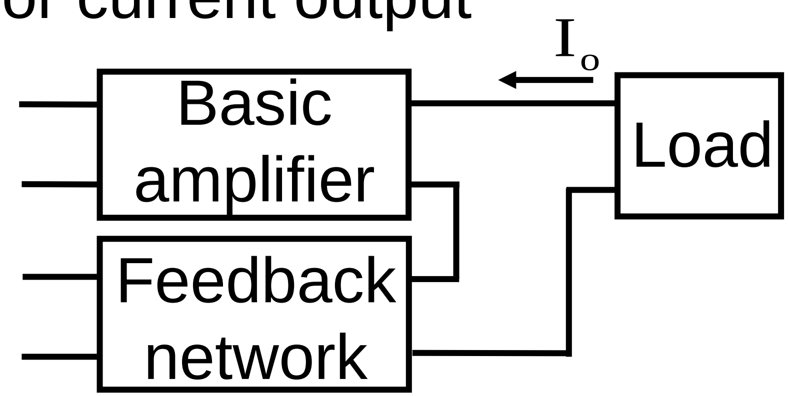
Single-Loop Feedback Amplifiers (Cont.)

- Sampling network
  - For voltage output



Voltage sampler

  - For current output

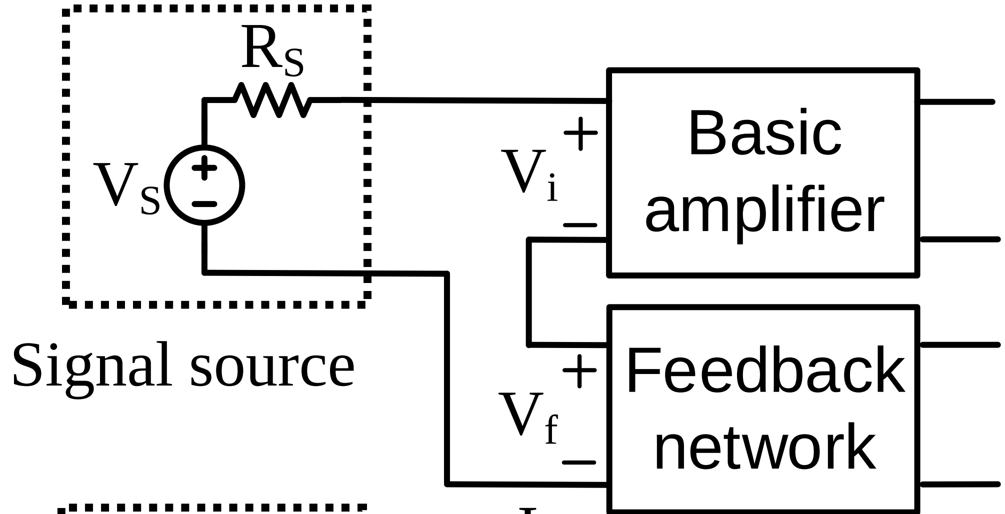


Current sampler

Single-Loop Feedback Amplifiers (Cont.)

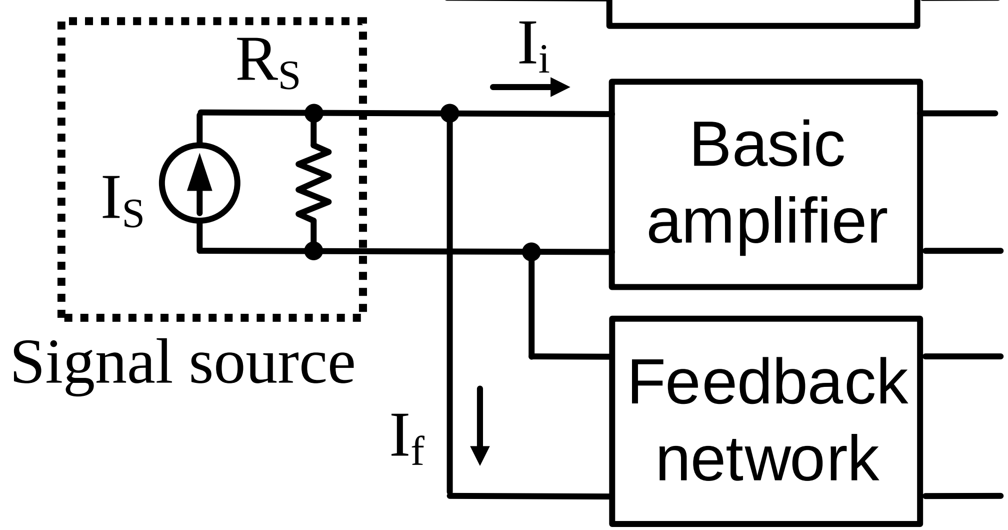
- Comparison or summing network
  - For voltage input

Voltage comparison



  - For current input

Current comparison



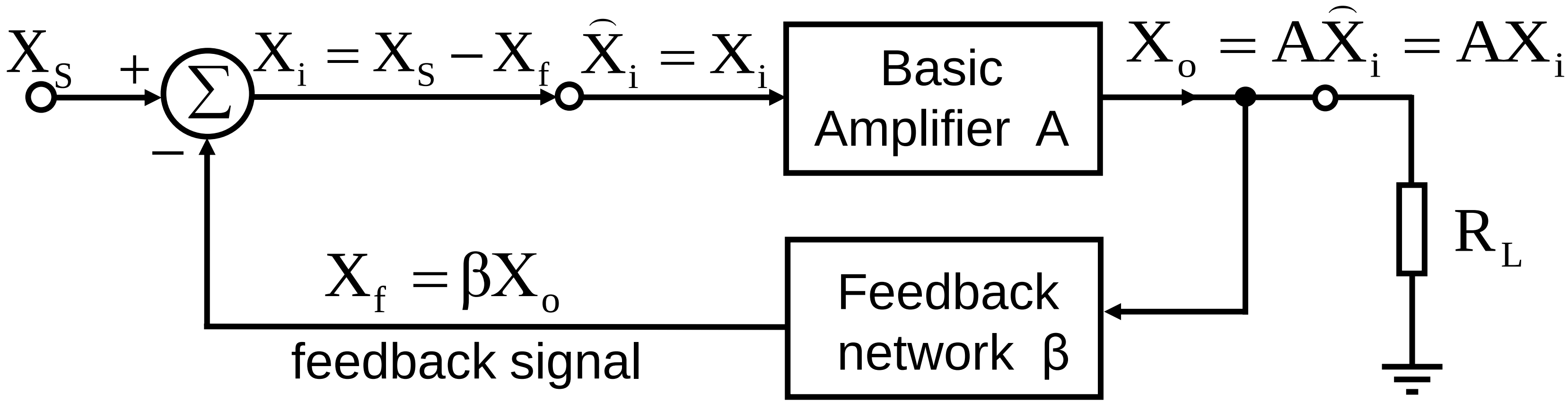
Single-Loop Feedback Amplifiers (Cont.)

- Feedback network
  - Usually, it's a passive network which may contain resistors, capacitors, and inductors.
- Basic amplifier
  - 4 types

    VCVS, CCCS, VCCS, CCVS
- 4 basic topologies
  - Series-shunt : for voltage amplifiers
  - Shunt-series : for current amplifiers
  - Series-series: for transconductance amplifiers
  - Shunt-shunt : for transimpedance amplifiers

Ideal Feedback Amplifier

- Block diagram



X_i: difference signal, $\hat{X}_i$: amplifier input

- ◆ Assumptions
  1. Feedback network is unilateral (i.e. output to input)
  2. Amplifier A is unilateral (i.e. input to output)
  3. Feedback factor (or transfer ratio) β is independent of the source and load resistance R_S and R_L

Ideal Feedback Amplifier (Cont.)

- Feedback factor (or transfer ratio)

$$\beta = \frac{X_f}{X_o}$$

- Open–loop gain

$$A = \frac{X_O}{\hat{X}_i} = \frac{X_O}{X_i} \quad \text{where} \quad X_i = X_S - X_f$$

- Close–loop gain

$$A_f = \frac{X_o}{X_s} = \frac{A}{1 + \beta A}$$

  - If $|A_f| < |A|$ → negative feedback
  - If $|A_f| > |A|$ → positive (or regenerative) feedback

Ideal Feedback Amplifier (Cont.)

- Return ratio (or Loop gain)

  T = βA

- Signals in feedback amplifier

Signal or Ratio	Feedback topology			
	Shunt-shunt	Shunt-series	Series-series	Series-shunt
X_o	Voltage	Current	Current	Voltage
$X_i X_s X_f$	Current	Current	Voltage	Voltage
	V_O / I_i	I_O / I_i	I_O / V_i	V_O / V_i
β	I_f / V_O	I_f / I_O	V_f / I_O	V_f / V_O

Properties of Negative-Feedback Amplifiers

- Sensitivity S_X^G

$S_X^G \equiv \dfrac{\Delta G / G_i}{\Delta X / X}$ where G is performance and X is component value

If $\dfrac{\Delta X}{X} << 1$, then $S_X^G \approx \dfrac{X}{G}\dfrac{dG}{dX}$

- For a feedback amplifier $S_T^{A_f} \approx \dfrac{T}{A_f}\dfrac{dA_f}{dT}$ and $S_A^{A_f} \approx \dfrac{A}{A_f}\dfrac{dA_f}{dA}$

$$A_f = \frac{A}{1+\beta A} = \frac{\beta}{\beta} \times \frac{A}{1+\beta A} = \frac{1}{\beta} \times \frac{\beta A}{1+\beta A} = K \times \frac{T}{1+T}$$

where K= $\dfrac{1}{\beta}$

If T changes by ΔT

$$\Delta A_f = \frac{K(T+\Delta T)}{1+T+\Delta T} - \frac{KT}{1+T} = \frac{K\Delta T}{(1+T+\Delta T)(1+T)}$$

$$\Rightarrow \quad S_T^{A_f} = \frac{1}{1+T+\Delta T} \approx \frac{1}{1+T} \qquad \text{if } |\Delta T| << |T|$$

Properties of Negative-Feedback Amplifiers (Cont.)

- Example: $T=49 \,\&\, \Delta T = +25$

$$\frac{\Delta A_f}{A_f} = \frac{\Delta T}{T} S_T^{A_f} = \frac{25}{49} \times \frac{1}{75} = 0.0068$$

- Example: $T = 49 \,\&\, \Delta T = -25$

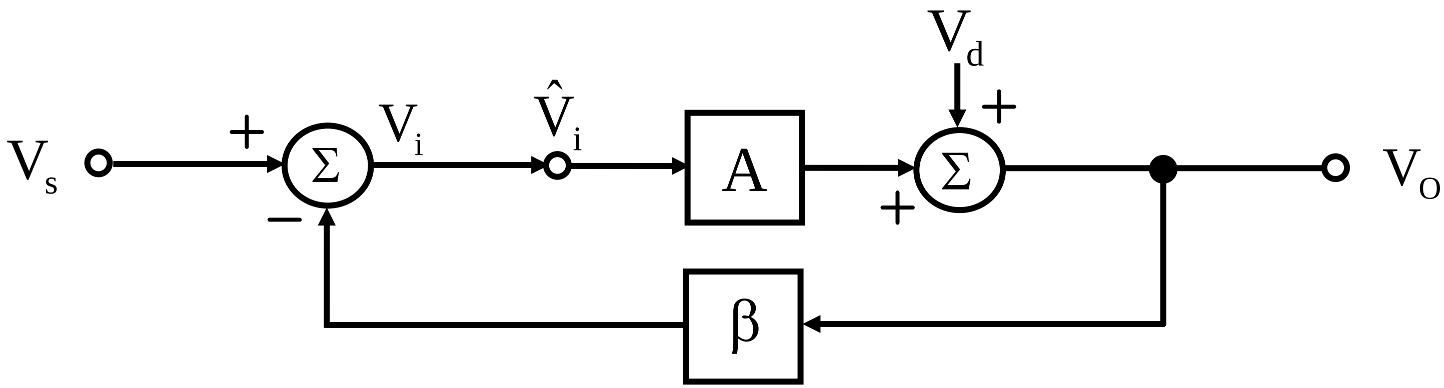
$$\frac{\Delta A_f}{A_f} = \frac{25}{49} \times \frac{1}{25} \approx 0.02$$

- For $T >> 1 \Rightarrow \quad \frac{T}{1+T} \approx 1 \quad \Rightarrow \quad A_f \approx \frac{1}{\beta}$

$\Rightarrow$ A_f is essentially independent of the gain of the basic amplifier and depends only on the ratio of passive components

Properties of Negative-Feedback Amplifiers (Cont.)

- Noise reduction



$$V_o = \frac{A}{1+\beta A} V_S + \frac{1}{1+\beta A} V_d$$

where V_d is generated due to the nonlinearity of A

Properties of Negative-Feedback Amplifiers (Cont.)

- Bandwidth extension
  - Open loop

$$A(s) = \frac{A_O}{1 + s/\omega_H}$$

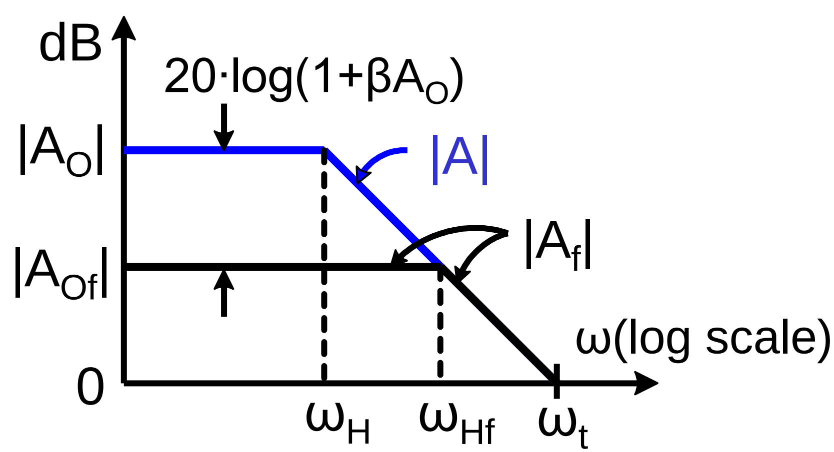
  - Close loop

$$A_f(s) = \frac{A(s)}{1 + \beta A(s)} = \frac{\dfrac{A_O}{1 + s/\omega_H}}{1 + \dfrac{\beta A_O}{1 + s/\omega_H}} = \frac{\dfrac{A_O}{1 + \beta A_O}}{1 + \dfrac{s}{\omega_H(1 + \beta A_O)}} = \frac{A_{Of}}{1 + s/\omega_{Hf}}$$

Close-loop gain $A_{Of} = \frac{A_O}{1 + \beta A_O}$

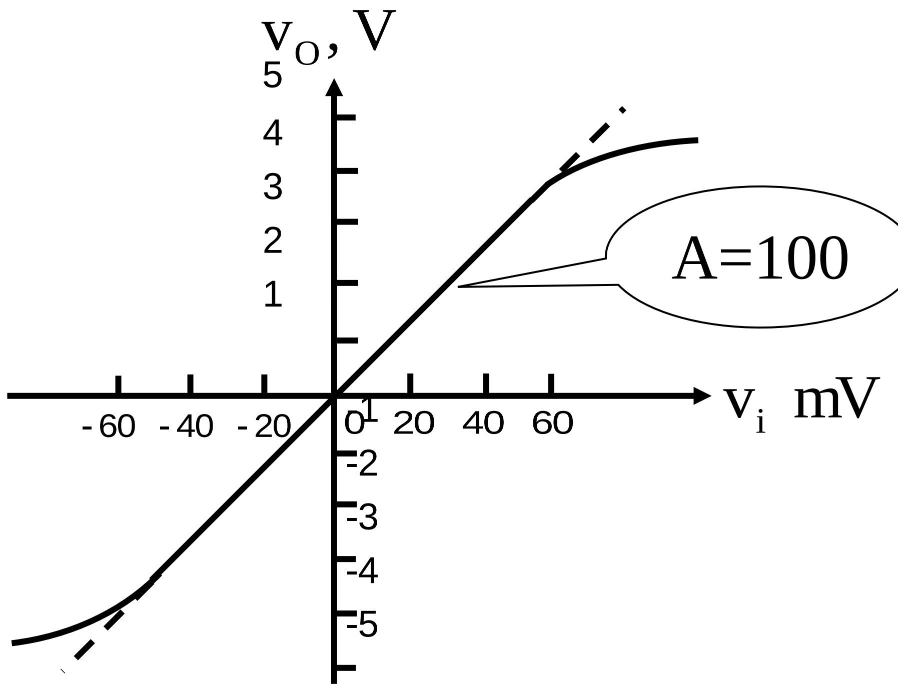
3-dB frequency $\omega_{Hf} = \omega_H(1 + \beta A_O)$

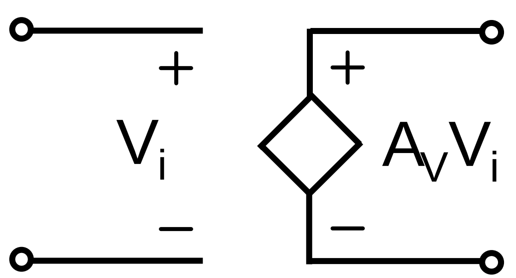
Unity-gain frequency ω_t



Properties of Negative-Feedback Amplifiers (Cont.)

- Nonlinear distortion
  - Without feedback





Properties of Negative-Feedback Amplifiers (Cont.)

$$\left|V_O\right| = 100\left|V_i\right| \;; \qquad 0 \le V_i \le 40\text{mV}$$

$$\left|V_O\right| = 100\left|V_i\right| - 2500(\left|V_i\right| - 0.04)^2 \;; \qquad 40 \le V_i \le 60\text{mV}$$

$$\left|V_O\right| = 5 \;; \qquad 60\text{mV} \le V_i$$

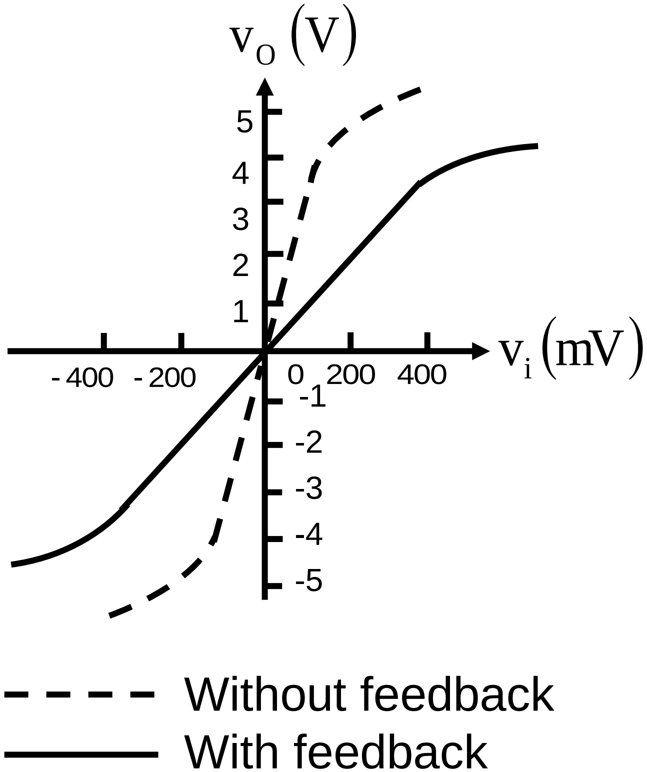
$\left\|V_o\right\|, (V)$	1.0	2.0	4.0	4.44	4.75	4.94	5.0
$\left\|V_s\right\| = \left\|V_i\right\|, (mV)$	10	20	40	45	50	55	60

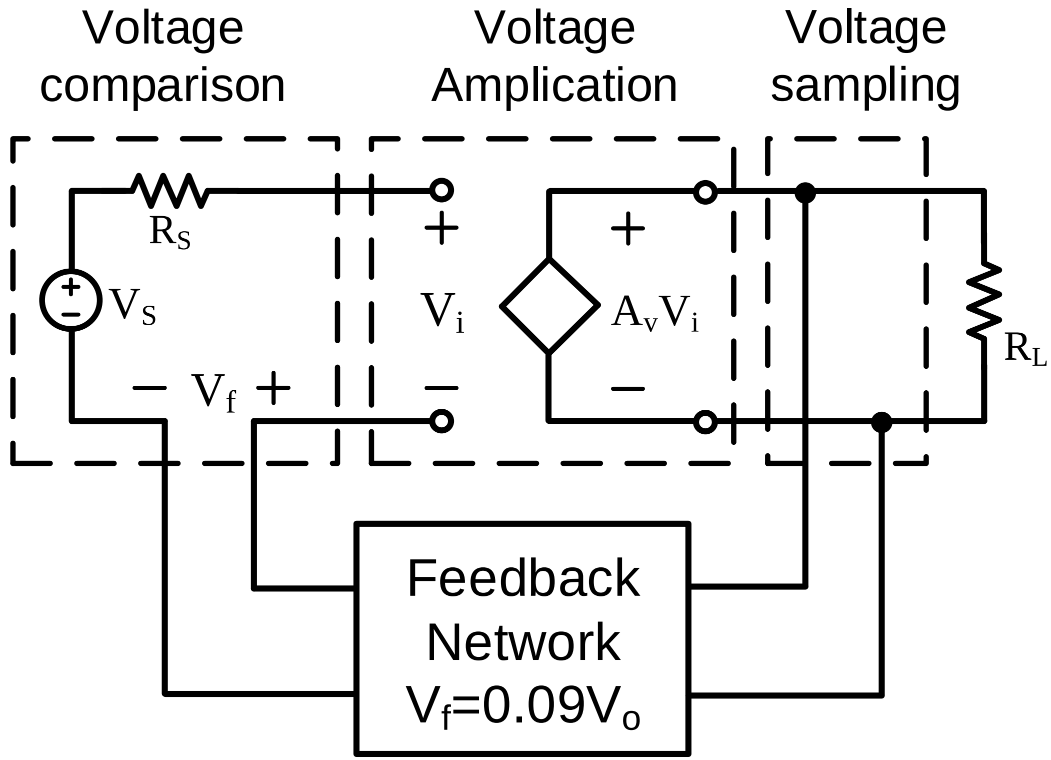
- With feedback $\beta = 0.09$

$$V_i = V_s - V_f \Rightarrow V_s = V_i + V_f = V_i + 0.09V_o$$

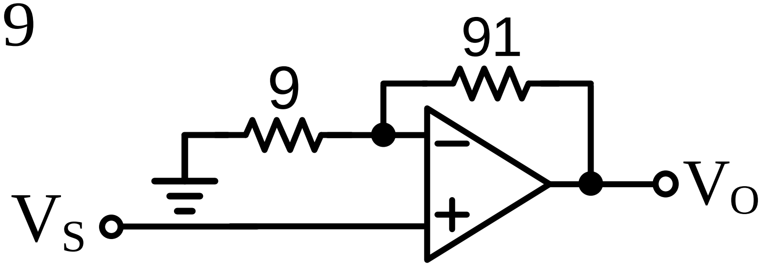
$\left\|V_o\right\|, (V)$	1.0	2.0	4.0	4.44	4.75	4.94	5.0
$\left\|V_s\right\|, (mV)$	100	200	400	444	478	500	510
$\left\|V_i\right\|, (mV)$	10	20	40	45	50	55	60

Properties of Negative-Feedback Amplifiers (Cont.)



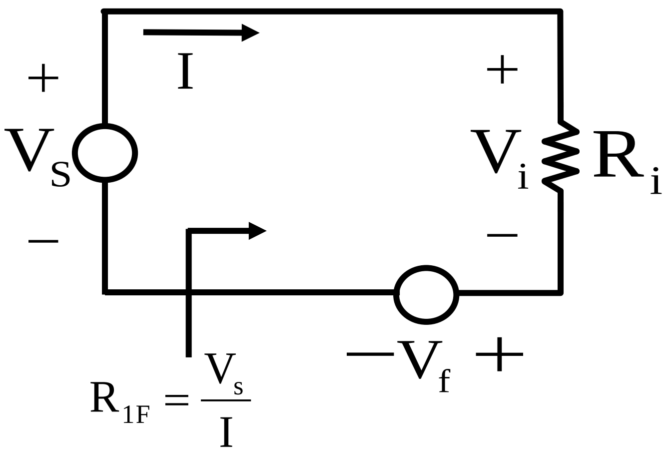


- e.g. $\beta = 0.09$



Properties of Negative-Feedback Amplifiers (Cont.)

- Input resistance R_{IF} (ideal situation)
  - Series connection (i.e. voltage input)



$$V_i = V_S - V_f \ ; \ V_f = \beta X_O = \beta A V_i \ ;$$

$$V_i = IR_i = \frac{V_O}{A} = \left(\frac{A}{1+\beta A} V_S \right) \frac{1}{A} = \frac{V_S}{1+\beta A}$$

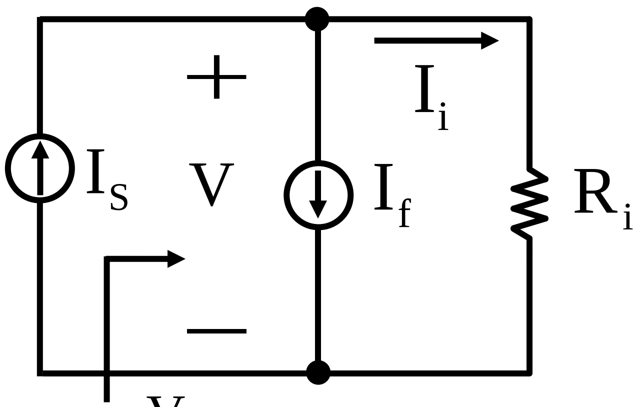
where X_o is either voltage output V_o or current output I_o and $A = A_v$ or G_m

$$\Rightarrow R_{IF} = \frac{V_S}{I} = R_i(1+\beta A) = R_i(1+T)$$

- For negative feedback, T > 0, input resistance is increased.

Properties of Negative-Feedback Amplifiers (Cont.)

◆ Shunt connection (i.e. current input)



$$I_i = I_S - I_f \quad ;$$

$$I_f = \beta X_O \;\&\; X_O = AI_i$$

$$R_{IF} = \frac{V}{I_S}$$

where X_o is either voltage output V_o or current output I_o and A =A_i or Z_m

$$\Rightarrow R_{IF} = \frac{V}{I_s} = R_i/(1+\beta A) = R_i/(1+T)$$

➢ For negative feedback, T > 0, input resistance is reduced.

Properties of Negative – Feedback Amplifiers (Cont.)

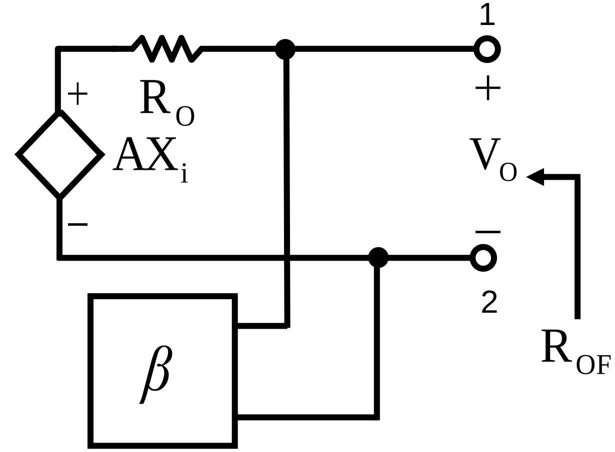
- Output resistance (ideal situation)

  Thevenin's theorem:

  Output resistance equals the ratio of open-circuit voltage to short-circuit current

  - Shunt connection (i.e. voltage output)
    - open-circuit voltage V_o

$$V_o = \frac{A}{1+\beta A} X_s$$



    - short-circuit current I_{sc}

$$I_{sc} = \frac{AX_i}{R_o} = \frac{AX_s}{R_o}$$

$$V_O = 0 \Rightarrow X_f = 0 \Rightarrow X_i = X_s$$

$$\Rightarrow R_{OF} = \frac{V_O}{I_{SC}} = \frac{R_O}{1+\beta\, A} = \frac{R_O}{1+T}$$

Properties of Negative – Feedback Amplifiers (Cont.)

- Series connection (i.e. current output)
  - short-circuit current I_o

$$I_o = -I_{SC} = \frac{A}{1+\beta A} X_S$$

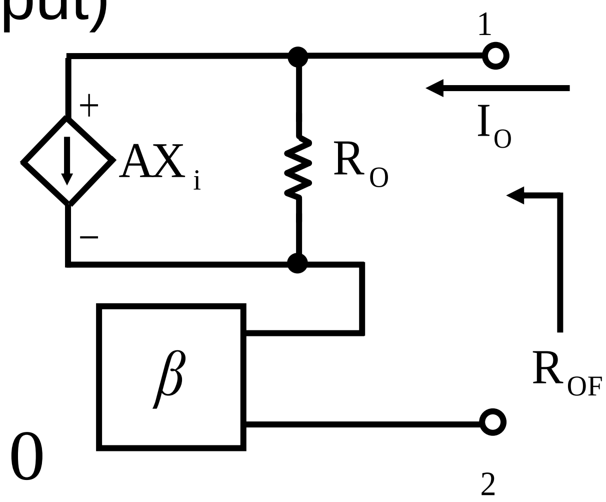
  - open-circuit voltage V_o

since $I_o = 0$ (open-circuit), $X_f = 0$

$$\Rightarrow X_i = X_S$$

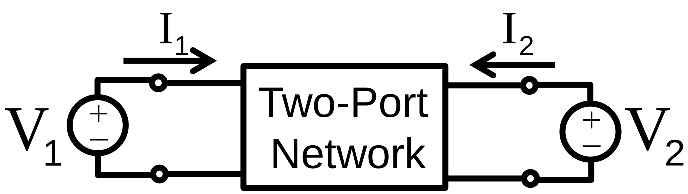
$$\Rightarrow V_{OC} = -AX_i R_O = -AX_S R_O$$

$$\Rightarrow R_{OF} = R_O(1+\beta A) = R_O(1+T)$$



Characterization of Linear Two-Port Network

- 4 ways to characterize two-port networks
  - y parameters

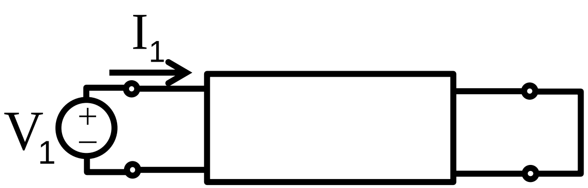


$$I_1 = y_{11}V_1 + y_{12}V_2$$

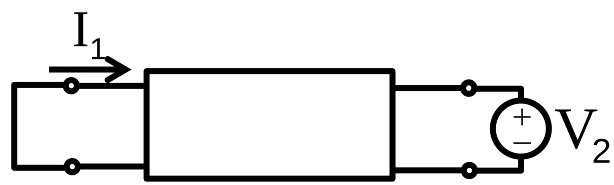
$$I_2 = y_{21}V_1 + y_{22}V_2$$

v_1 and v_2 are independent variables (excitation)

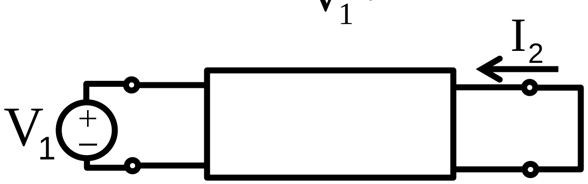
I_1 and I_2 are dependent variables (response)



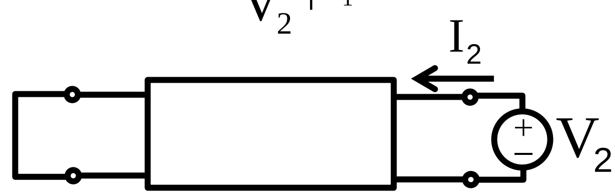
$$y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0}$$



$$y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0}$$



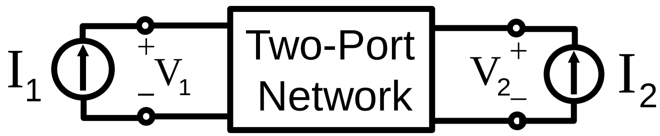
$$y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0}$$



$$y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0}$$

Characterization of Linear Two-Port Network (Cont.)

◆ z parameters



$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

I_1 and I_2 are independent variables (excitation)

v_1 and v_2 are dependent variables (response)

$$z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0}$$

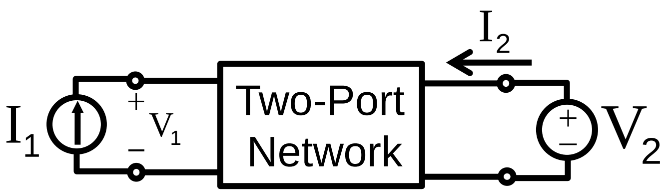
$$z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0}$$

$$z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}$$

$$z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0}$$

Characterization of Linear Two-Port Network (Cont.)

- h parameters



$$V_1 = h_{11}I_1 + h_{12}V_2$$

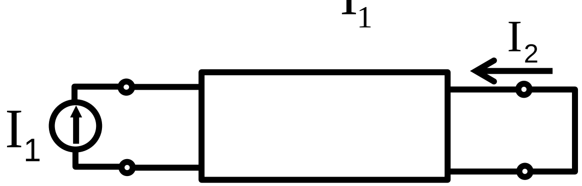
$$I_2 = h_{21}I_1 + h_{22}V_2$$

I_1 and v_2 are independent variables (excitation)

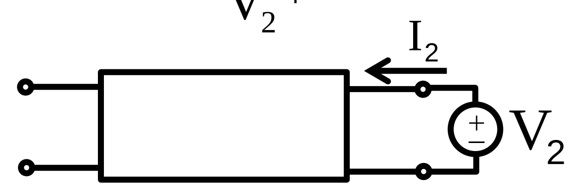
v_1 and I_2 are dependent variables (response)

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$



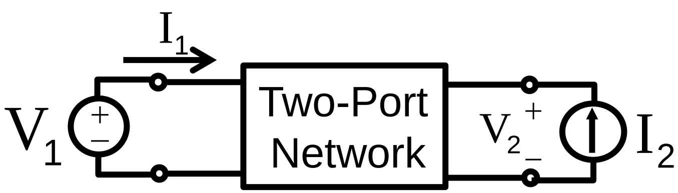
$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$



$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

Characterization of Linear Two-Port Network (Cont.)

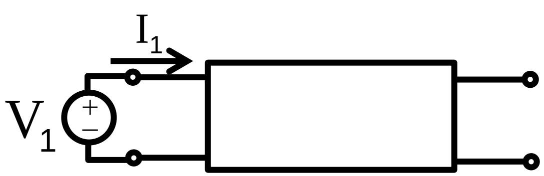
- g parameters



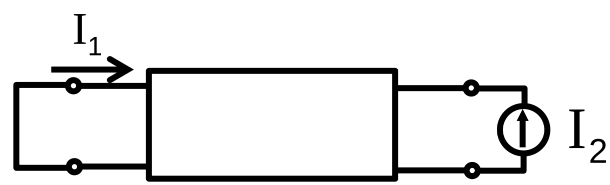
$$I_1 = g_{11}V_1 + g_{12}I_2$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

- v_1 and I_2 are independent variables (excitation)
- I_1 and v_2 are dependent variables (response)



$$g_{11} = \frac{I_1}{V_1}\Big|_{I_2=0}$$



$$g_{12} = \frac{I_1}{I_2}\Big|_{V_1=0}$$

$$g_{21} = \frac{V_2}{V_1}\Big|_{I_2=0}$$

$$g_{22} = \frac{V_2}{I_2}\Big|_{V_1=0}$$

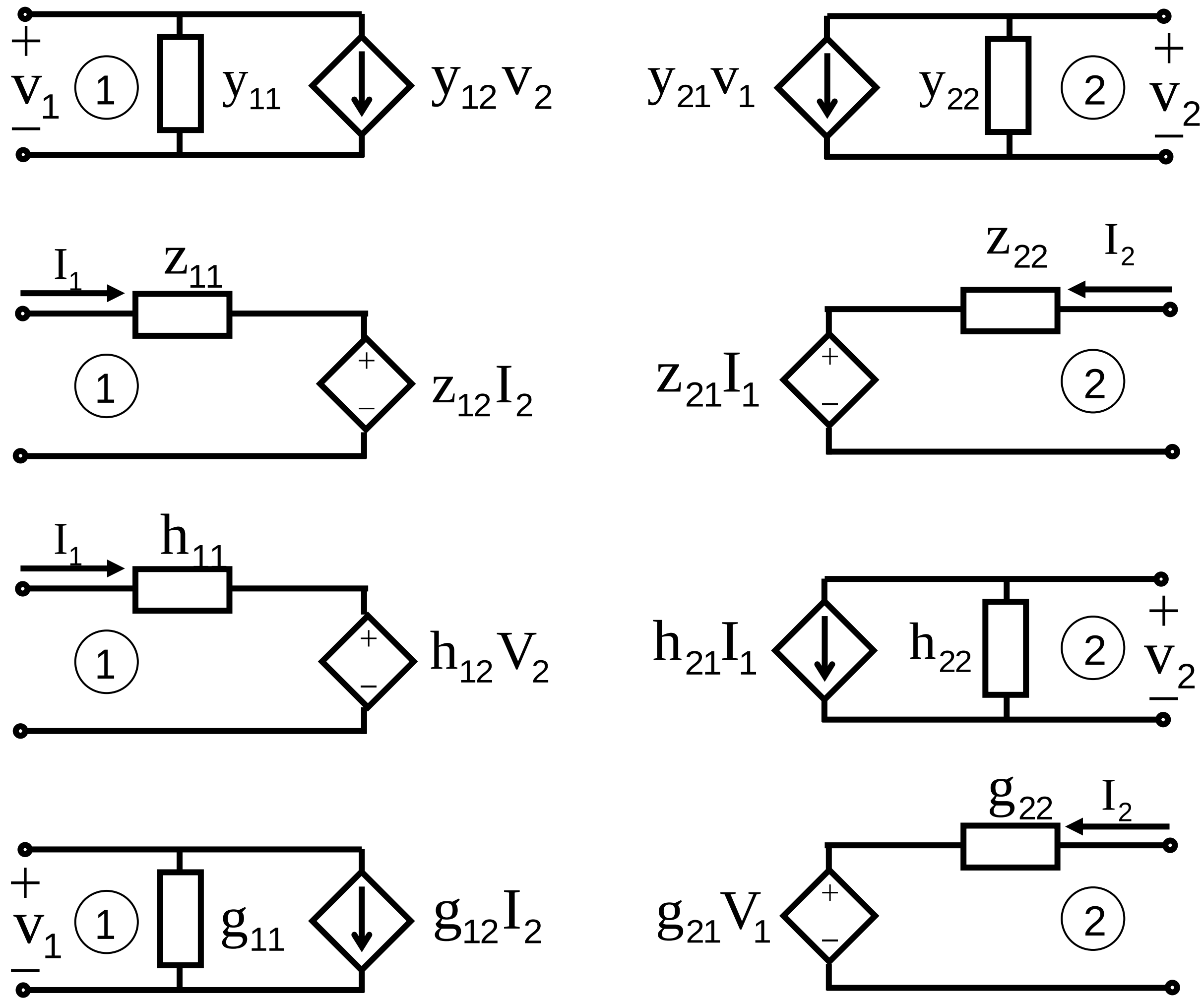
Equivalent Circuits for y, z, h, and g Parameters

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

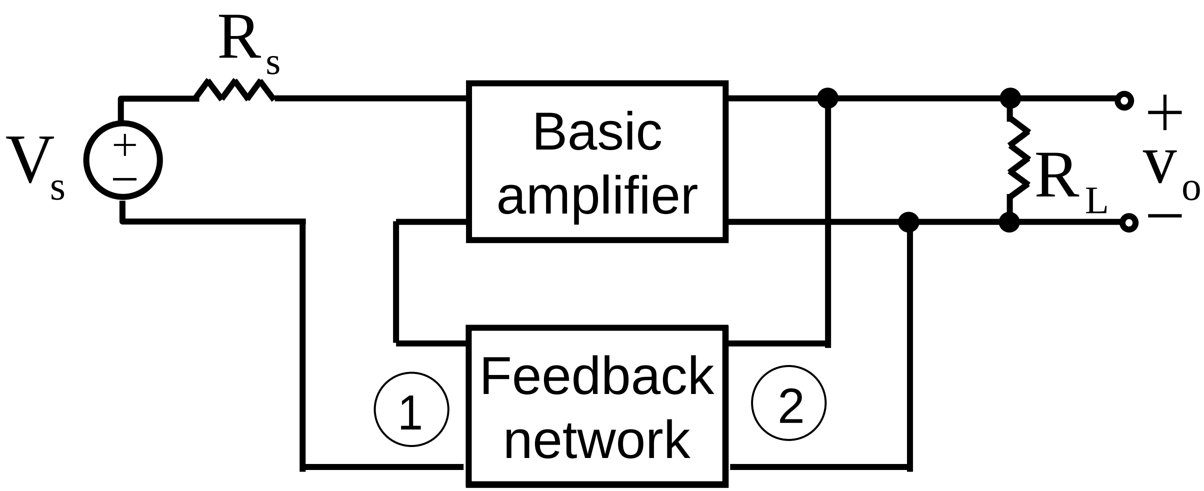
$$I_1 = g_{11}V_1 + g_{12}I_2$$
$$V_2 = g_{21}V_1 + g_{22}I_2$$



Voltage Amplifier with Feedback

- Series-shunt type
- Basic amp
  - Input voltage: V_s in series with R_s
  - Output voltage: V_o in parallel with R_L

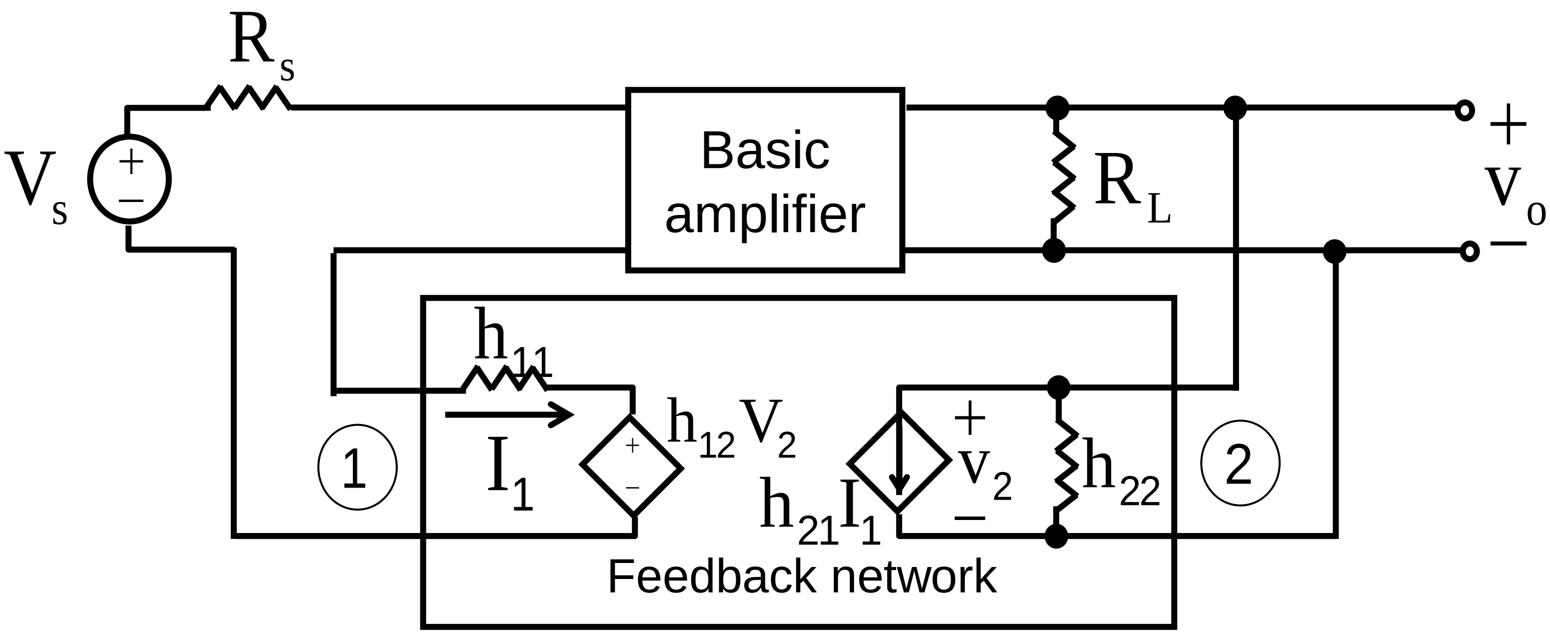
⇨ Feedback network characterize using h-parameters



Voltage Amplifier with Feedback (Cont.)

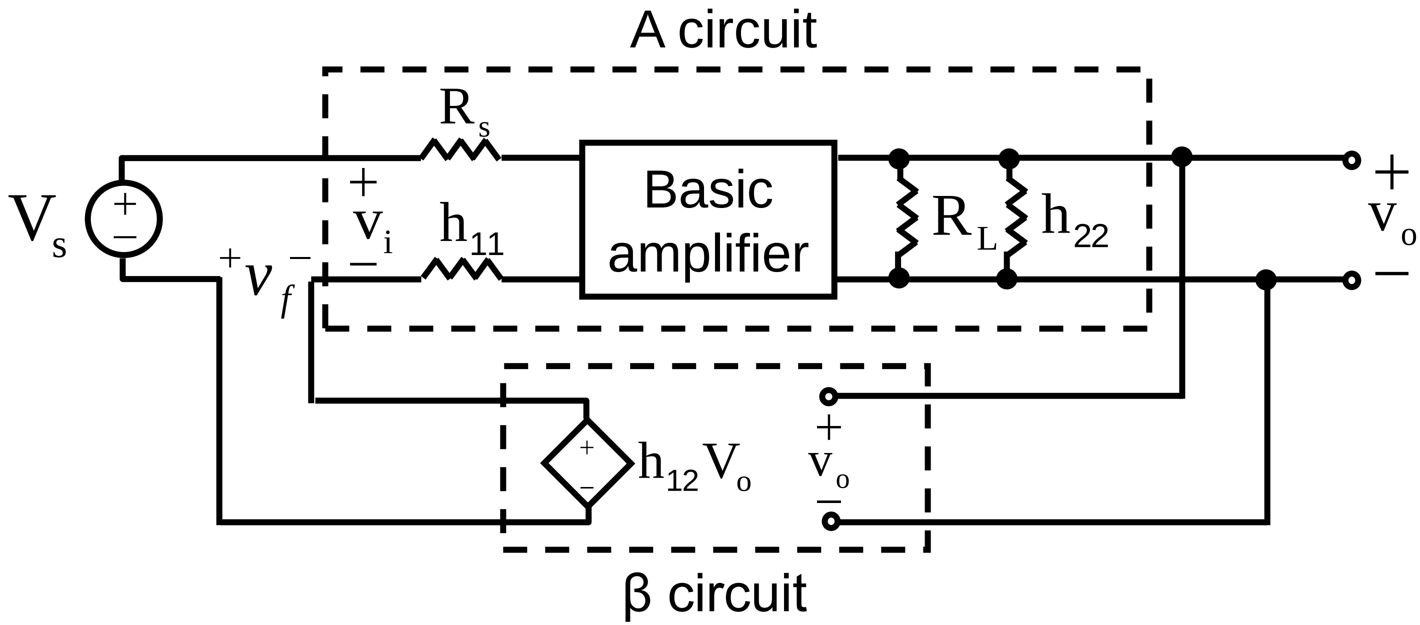
- Unilateral basic amp
  Unilateral feedback network } are assumed

⇨ $\left|h_{21}\right|_{feedback\_network}$ can be neglected



Voltage Amplifier with Feedback (Cont.)

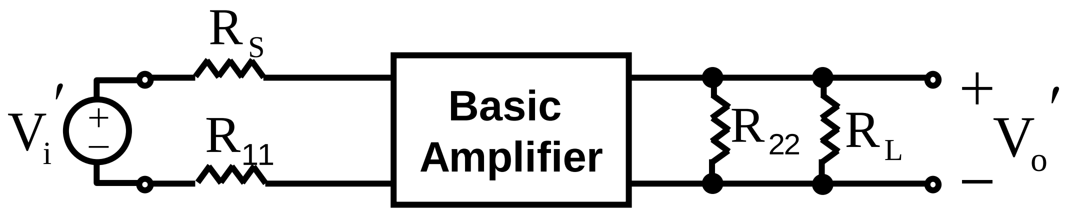
- $\boldsymbol{\beta = h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0}}$



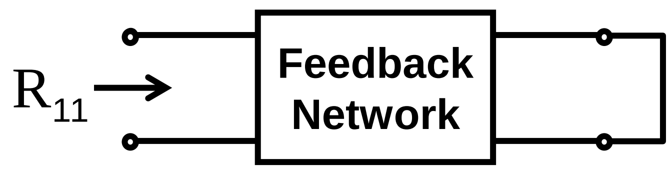
A and β Circuit of a Voltage Amplifier with Feedback

- Summary of the rules for finding A and β
  - "A" Circuit

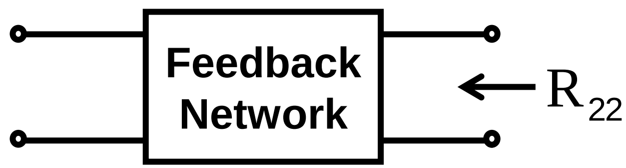
$$A = \frac{V_o'}{V_i'}$$



(1) R_{11} is obtained from

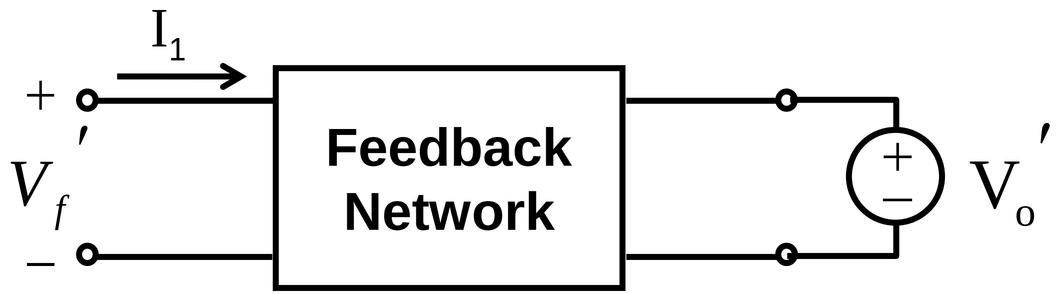


(2) R_{22} is obtained from



A and β Circuit of a Voltage Amplifier with Feedback (Cont.)

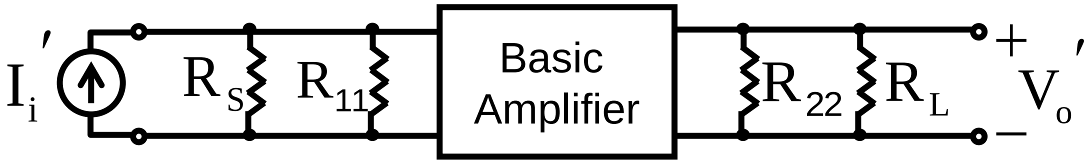
$$\beta = \left.\frac{V_f'}{V_o'}\right|_{I_1=0}$$



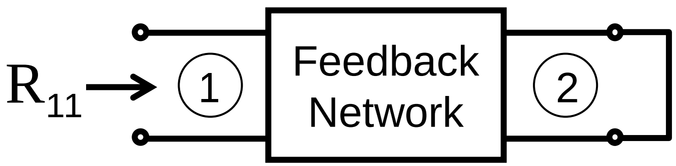
- Current Amplifier
  Transconductance Amplifier
  Transimpedance Amplifier
  } can be similarly obtained

A and β Circuit of a Transimpedance Amplifier with Feedback

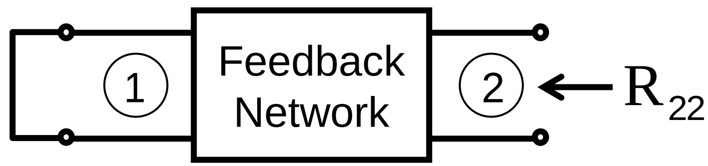
(a) The A circuit is



where R_{11} is obtain from

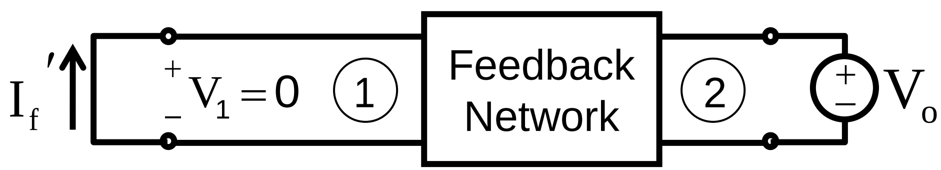


and R_{22} is obtain from



and the gain A is defined $A \equiv \frac{V_o'}{I_i'}$

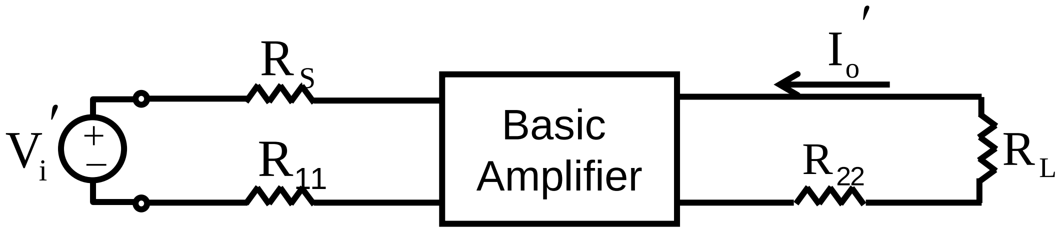
(b) β is obtained from



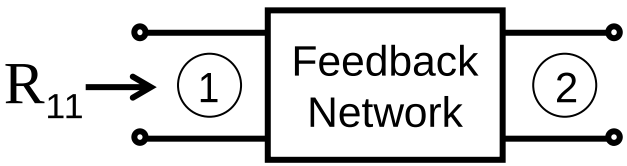
$$\beta = \left. \frac{I_f'}{V_o'} \right|_{V_1=0}$$

A and β Circuit of a Transconductance Amplifier with Feedback

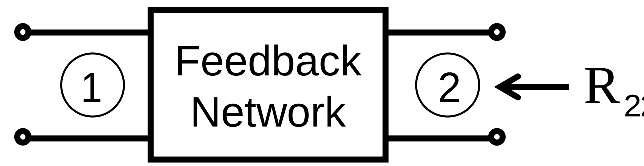
(a) The A circuit is



where R_{11} is obtained from

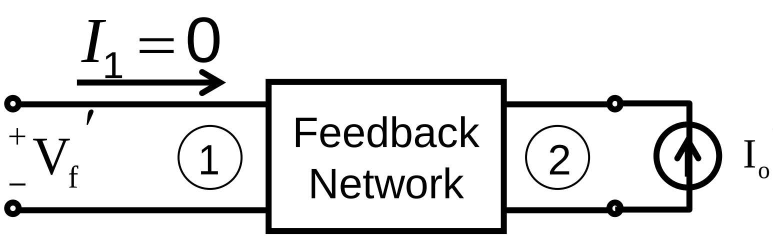


and R_{22} is obtained from



and the gain A is defined $A \equiv \dfrac{I_o'}{V_i'}$

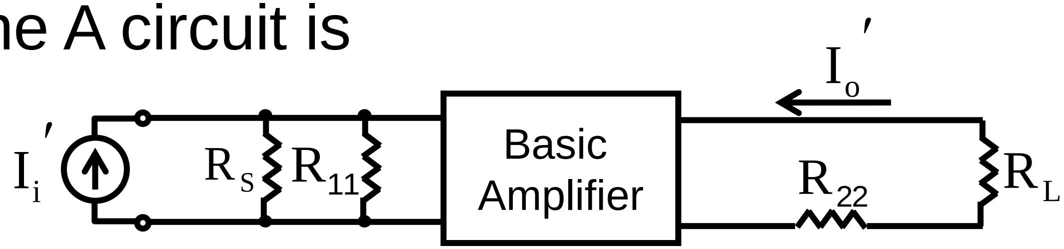
(b) β is obtained from



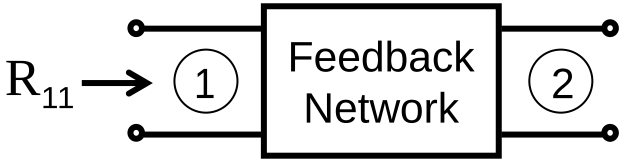
$$\beta = \left.\frac{V_f'}{I_o'}\right|_{I_1=0}$$

A and β Circuit of a Current Amplifier with Feedback

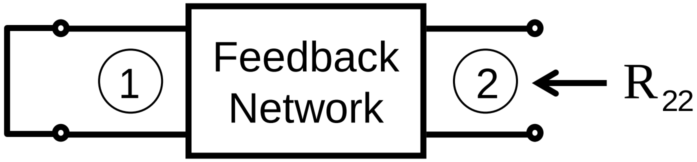
(a) The A circuit is



where R_{11} is obtain from

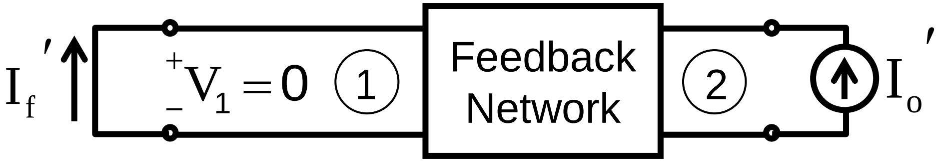


and R_{22} is obtain from



and the gain A is defined $A \equiv \frac{I_o'}{I_i'}$

(b) β is obtained from



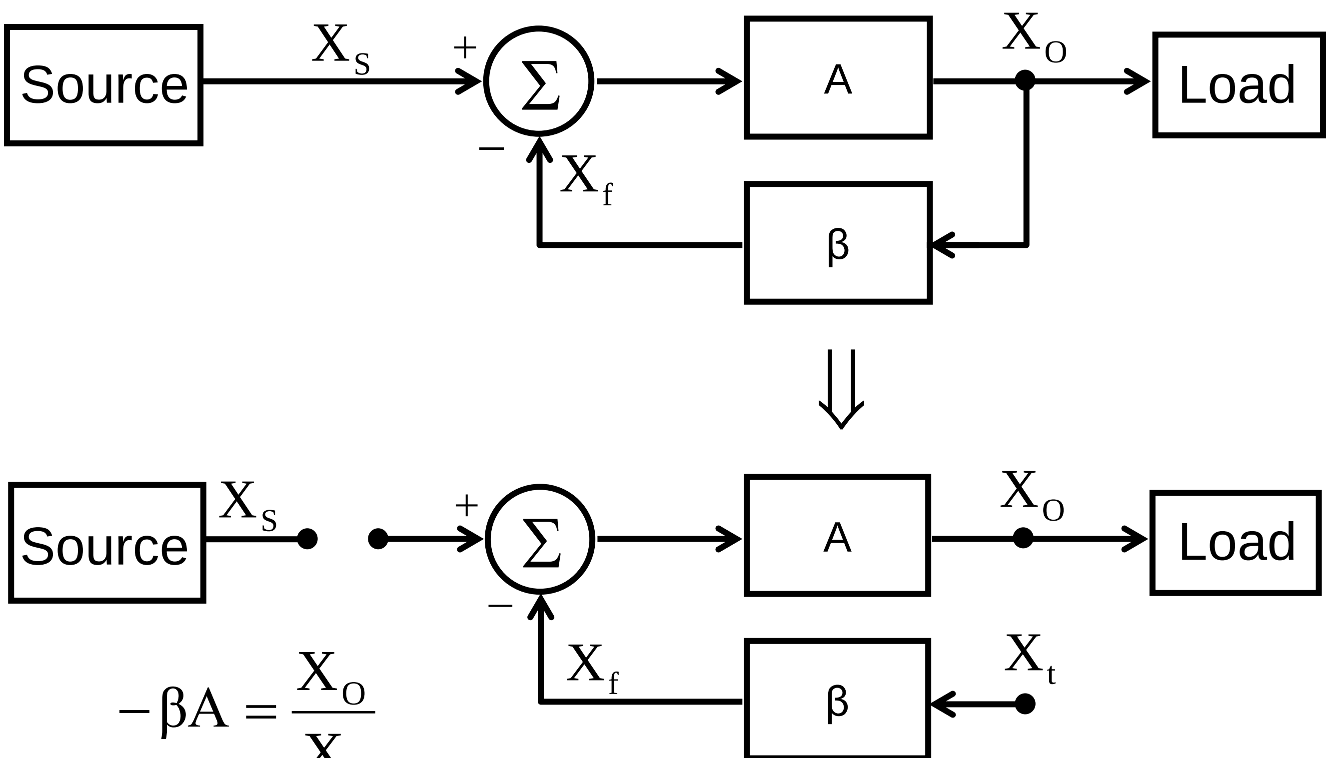
$$\beta = \left.\frac{I_f'}{I_o'}\right|_{V_1=0}$$

Summary of Relationship for the Four Feedback-Amplifier Topologies

Feedback Amplifier	X_i	X_O	X_f	X_S	A	β	A_f	Source Form	Loading of Feedback network is obtained: At input	Loading of Feedback network is obtained: At output	To Find β apply to port 2 of feedback network	Z_{if}	Z_{of}
Series-shunt (voltage amplifier)	V_i	V_O	V_f	V_S	$\frac{V_O}{V_i}$	$\frac{V_f}{V_O}$	$\frac{V_O}{V_S}$	Thevenin	By short-circuiting port2 of feedback network	By open-circuiting port1 of feedback network	A voltage and find the open-circuit voltage at port 1	$Z_i(1+A\beta)$	$\frac{Z_0}{1+A\beta}$
Shunt-series (current amplifier)	I_i	I_O	I_f	I_S	$\frac{I_O}{I_i}$	$\frac{I_f}{I_O}$	$\frac{I_O}{I_S}$	Norton	By open-circuiting port2 of feedback network	By short-circuiting port1 of feedback network	A current and find the short-circuit voltage at port 1	$\frac{Z_i}{1+A\beta}$	$Z_0(1+A\beta)$
Series-series (transconductance amplifier)	V_i	I_O	V_f	V_S	$\frac{I_O}{V_i}$	$\frac{V_f}{I_O}$	$\frac{I_O}{V_S}$	Thevenin	By open-circuiting port2 of feedback network	By open-circuiting port1 of feedback network	A current and find the open-circuit voltage at port 1	$Z_i(1+A\beta)$	$Z_0(1+A\beta)$
Shunt-shunt (transresistance amplifier)	I_i	V_O	I_f	I_S	$\frac{V_O}{I_i}$	$\frac{I_f}{V_O}$	$\frac{V_O}{I_S}$	Norton	By short-circuiting port2 of feedback network	By short-circuiting port1 of feedback network	A voltage and find the short-circuit voltage at port 1	$\frac{Z_i}{1+A\beta}$	$\frac{Z_0}{1+A\beta}$

Determining the Loop Gain

- Loop gain βA determines whether an feedback amplifier is stable or oscillatory
- Determination of loop gain
  - ◆ Break a connection line in the loop and disconnect input

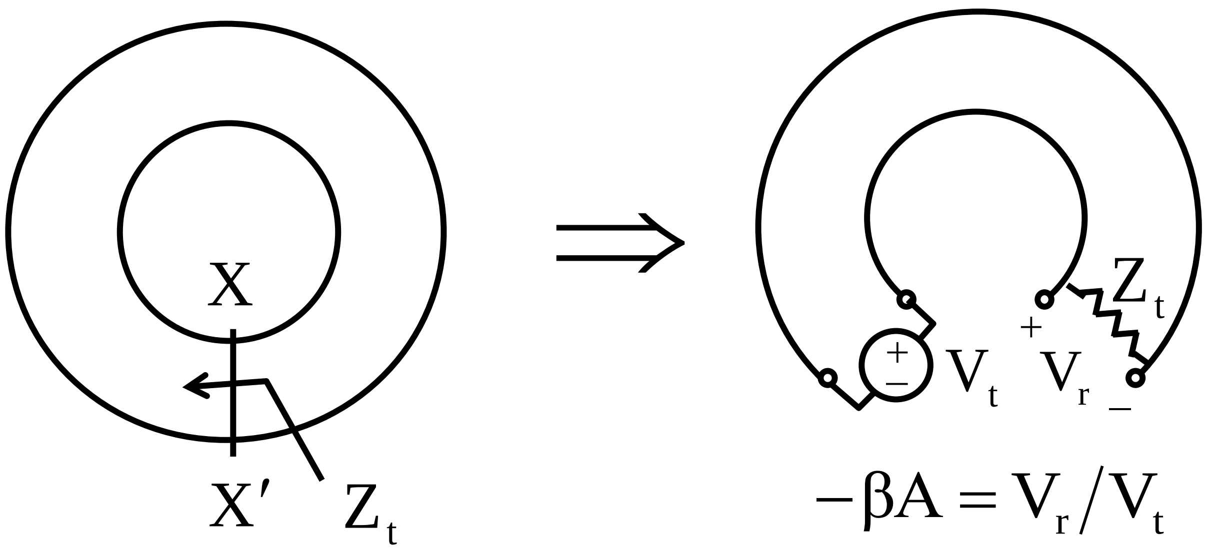


$$-\beta A = \frac{X_O}{X_t}$$

Determining the Loop Gain (Cont.)

- Conditions that existed prior to breaking the loop do not change.

By terminating the loop where it is opened with an impedance equal to that seen before the loop was broken.

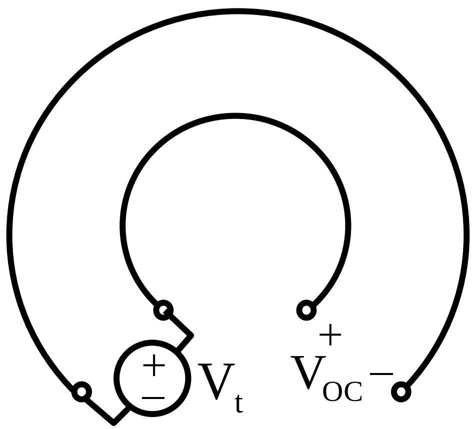


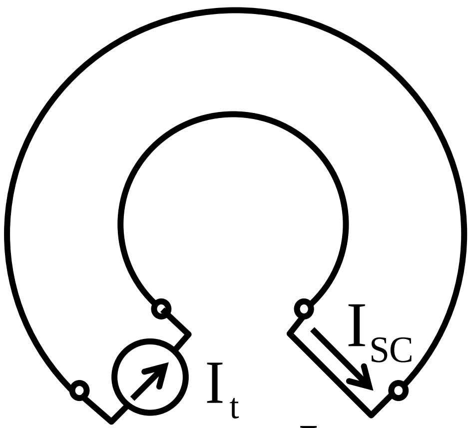
$$-\beta A = V_r / V_t$$

$$-\beta A = \frac{V_r}{V_t} \quad \text{or} \quad -\beta A = \frac{I_r}{I_t} \quad \text{if current is applied}$$

Determining the Loop Gain (Cont.)

- An alternative method for determining βA (See Rosenstark, 1986)
  - Find the open-circuit function T_{OC} and the short-circuit function T_{SC} and combine them
    - Open-circuit
    - Short-circuit





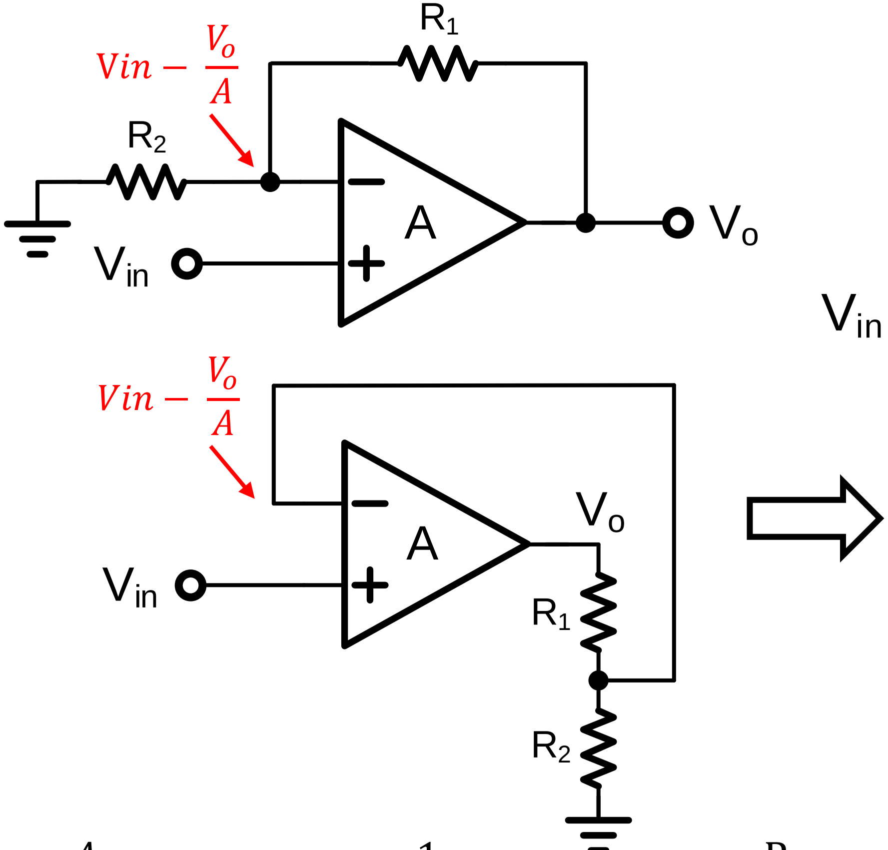
$$T_{OC} \equiv \frac{V_{OC}}{V_t}$$

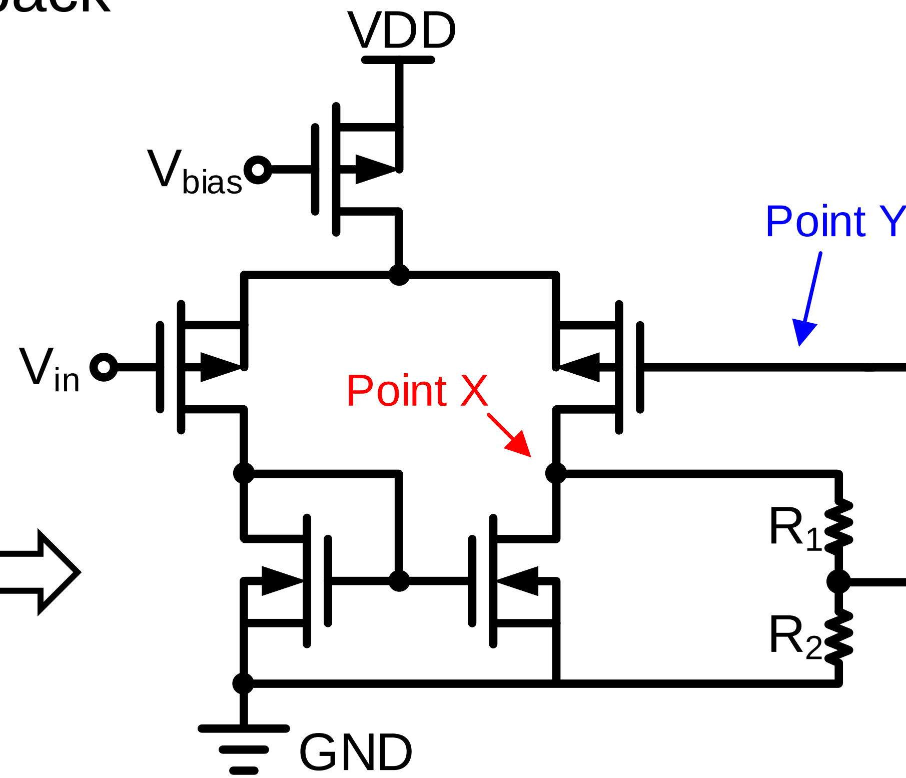
$$T_{SC} \equiv \frac{I_{SC}}{I_t}$$

$$\Rightarrow \quad -\beta A = \frac{1}{\dfrac{1}{T_{OC}} + \dfrac{1}{T_{SC}}}$$

Determining the Loop Gain (Cont.)

- OPAMP with negative feedback





$$V_o = \frac{A}{1+\beta A} V_{in} \Rightarrow V_o = \frac{1}{\beta} V_{in} \Rightarrow V_o = \left(1 + \frac{R_1}{R_2}\right) V_{in}$$

$A \neq \infty$ $\qquad$ $A = \infty$ $\qquad$ $\beta = \dfrac{R_2}{R_1 + R_2}$

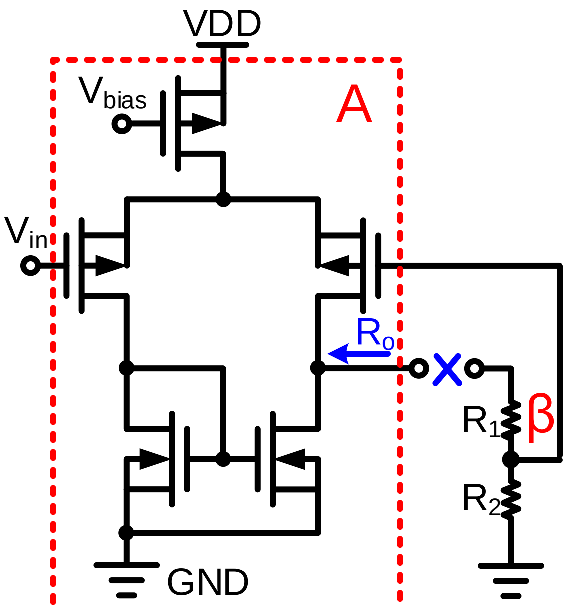
$$A = g_m R_o$$

$$R_o = r_{dsn} \parallel r_{dsp} \parallel (R_1 + R_2)$$

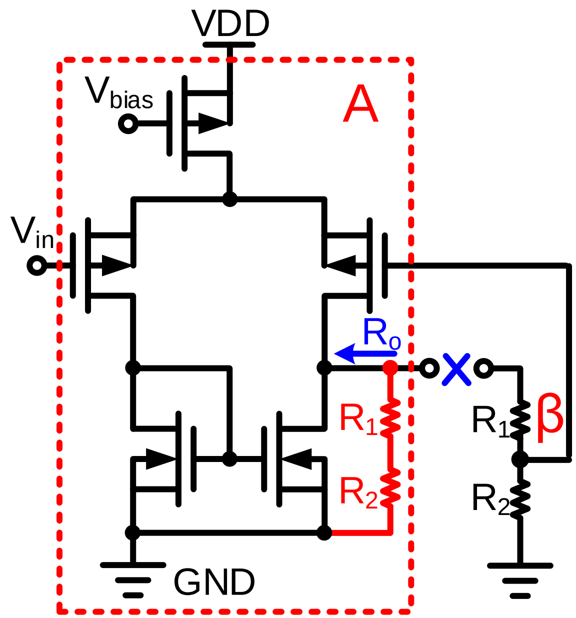
$$\beta = \frac{R_2}{R_1 + R_2}$$

Determining the Loop Gain (Cont.)

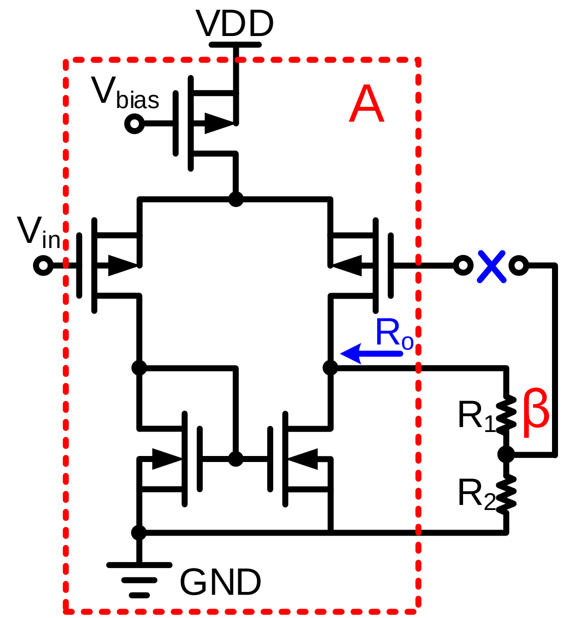
- βA analysis
  - Method 1 (break at **point X**)
  - Method 2 (break at **point Y**)



Wrong



Correct



Correct

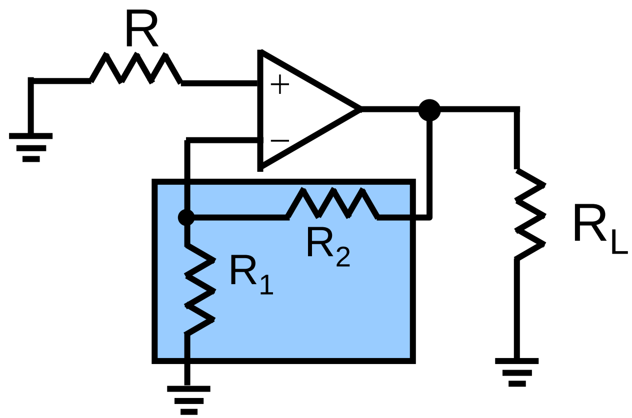
Wrong： $A = g_m(r_{dsn} \parallel r_{dsp})$

Correct： $A = g_m[r_{dsn} \parallel r_{dsp} \parallel (R_1 + R_2)]$

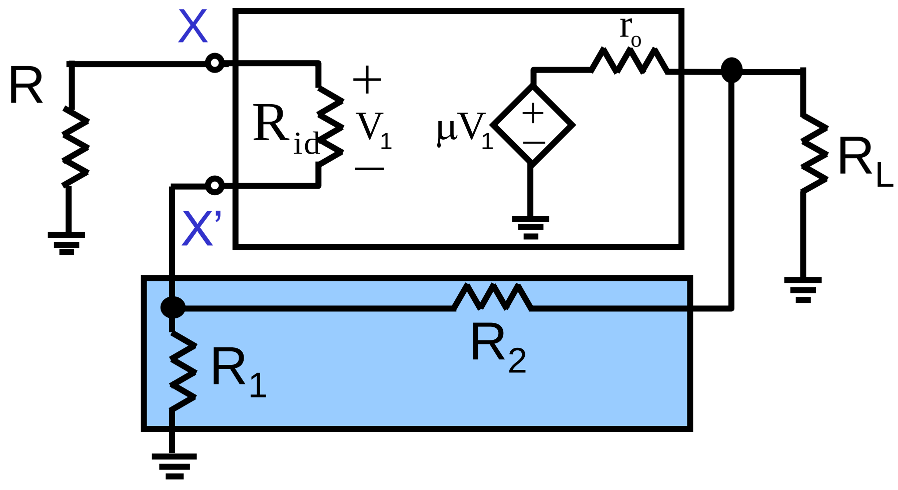
$$\beta = \frac{R_2}{R_1 + R_2}$$

Determining the Loop Gain (Cont.)

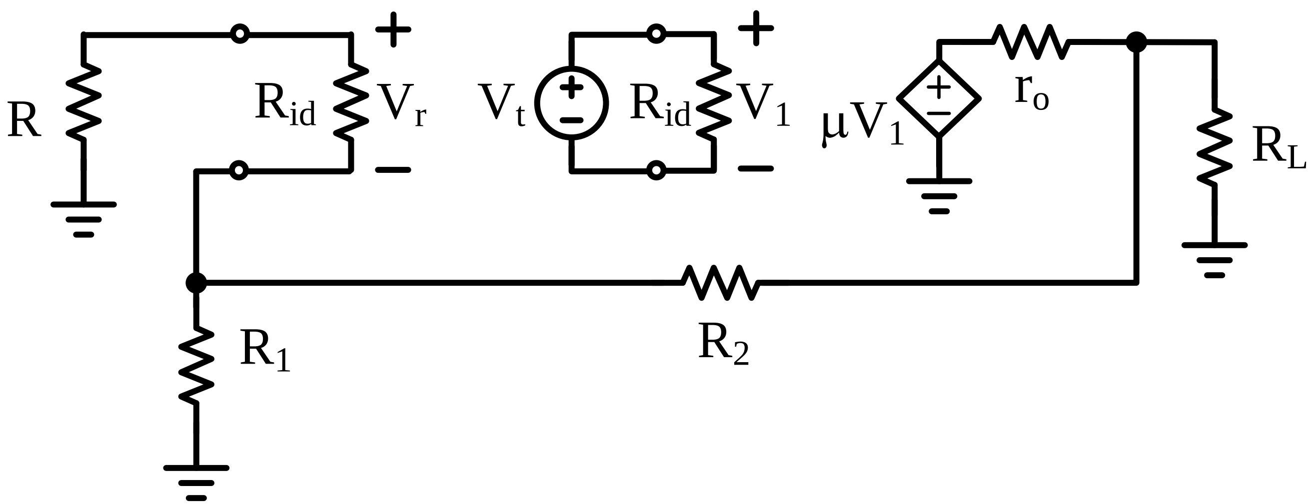
- Example for both inverting and noninverting
  - Schematic



  - Equivalent model



Determining the Loop Gain (Cont.)



- Loop gain

$$-\beta A=\frac{V_r}{V_t}=\frac{V_r}{V_1}=-\mu\times\frac{\{R_L//[R_2+R_1//(R_{id}+R)]\}}{\{R_L//[R_2+R_1//(R_{id}+R)]\}+r_o}\times\frac{[R_1//(R_{id}+R)]}{[R_1//(R_{id}+R)]+R_2}\times\frac{R_{id}}{R_{id}+R}$$

Stability and Response of Feedback Amplifiers

- Effect of feedback on bandwidth
  - One-pole amplifier

1. $A(s) = \dfrac{A_O}{1 + s/\omega_H}$ & $T(s) = \beta A(s) = \dfrac{\beta A_O}{1 + s/\omega_H}$

   where A_O is the midband values of A(s) and ω_H is the angular frequency of the dominant pole

2. $A_f(s) = \dfrac{A(s)}{1 + \beta A} = \dfrac{\dfrac{A_O}{1 + s/\omega_H}}{1 + \dfrac{\beta A_O}{1 + s/\omega_H}} = \dfrac{\dfrac{A_O}{1 + \beta A_O}}{1 + \dfrac{s}{\omega_H(1 + \beta A_O)}} = \dfrac{A_{Of}}{1 + s/\omega_{Hf}}$

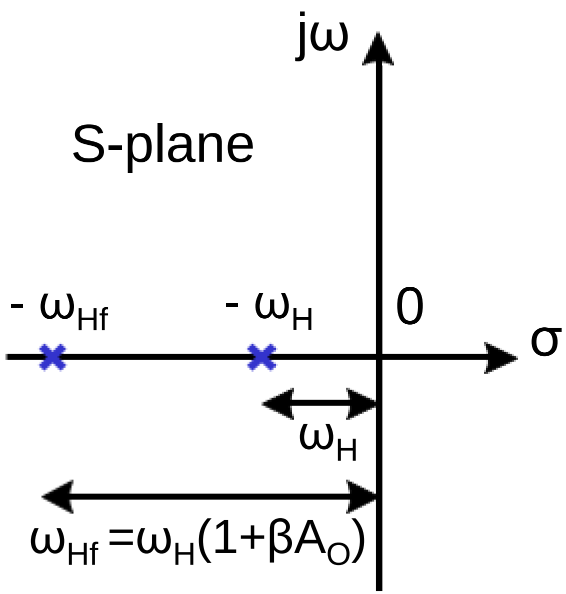
   where the midband closed-loop gain $A_{Of} = \dfrac{A_O}{1 + \beta A_O}$ and the 3-dB angular frequency $\omega_{Hf} = (1 + \beta A_O)\,\omega_H$

$\Rightarrow$ Negative feedback has increased the bandwidth by a factor $(1 + \beta A_O)$

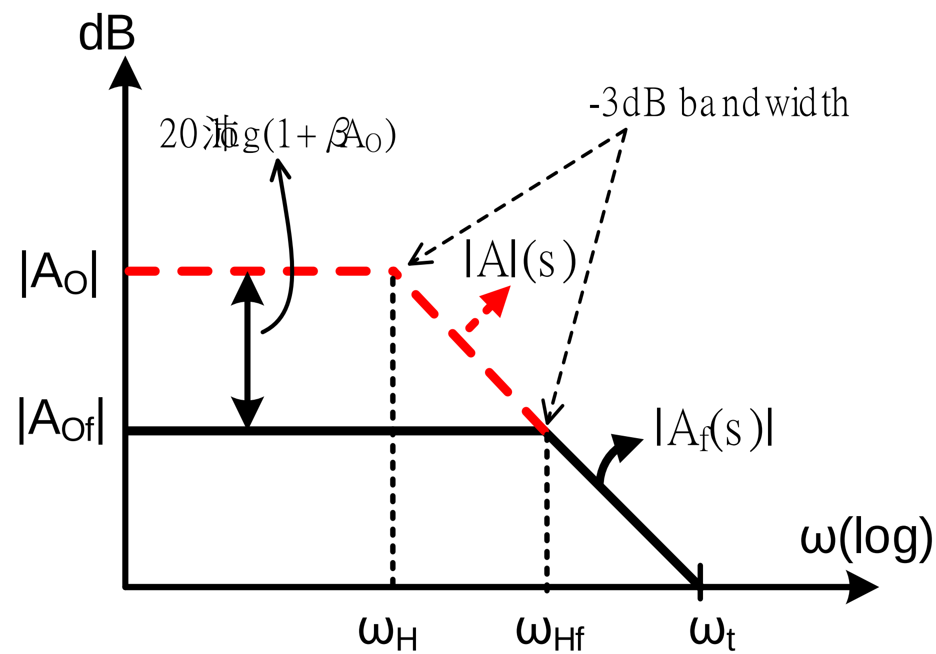
$\Rightarrow$ $A_O \omega_H = A_{Of} \omega_{Hf}$; Gain-Bandwidth products are equal.

Stability and Response of Feedback Amplifiers (Cont.)

- Pole location



- Frequency response



For one-pole system, unity-gain bandwidth (ω_t)

$\omega_t = A_O \omega_H = A_{Of} \omega_{Hf}$ =gain-bandwidth product

Stability and Response of Feedback Amplifiers (Cont.)

- Two-pole function

  Poles $s_1 = -\omega_1$ & $s_2 = -\omega_2$

$$A(s) = \frac{A_O}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)} = \frac{A_O}{1+s\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right)+\frac{s^2}{\omega_1\omega_2}}$$

$$T(s) = \frac{\beta A_O}{1+s\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right)+\frac{s^2}{\omega_1\omega_2}} = \frac{\beta A_O}{1+a_1 s+a_2 s^2}$$

where $\omega_1 \approx \frac{1}{a_1}$ & $\omega_2 \approx \frac{a_1}{a_2}$ if ω_1 & ω_2 are widely separated.

Stability and Response of Feedback Amplifiers (Cont.)

$$A_f(s) = \frac{A(s)}{1+\beta A(s)} = \frac{\dfrac{A_O}{1+\beta A_O}}{1+\dfrac{a_1 s}{1+\beta A_O}+\dfrac{a_2 s^2}{1+\beta A_O}}$$

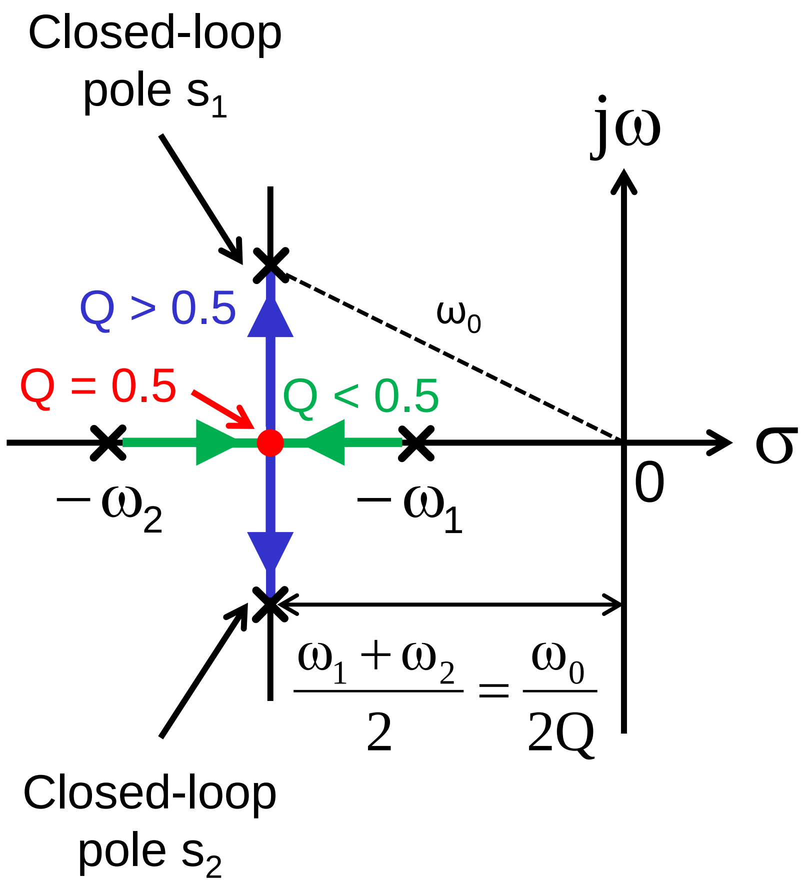
$$= \frac{A_{Of}}{1+\dfrac{s}{1+\beta A_O}\left(\dfrac{1}{\omega_1}+\dfrac{1}{\omega_2}\right)+\dfrac{s^2}{(1+\beta A_O)\omega_1\omega_2}}$$

$$= \frac{A_{Of}}{1+\left(\dfrac{s}{\omega_0}\right)\left(\dfrac{1}{Q}\right)+(\dfrac{s}{\omega_0})^2}$$

$$\omega_0 = \sqrt{\omega_1\omega_2(1+\beta A_0)} \;\; \& \;\; Q = \frac{\omega_0}{\omega_1+\omega_2}$$

Stability and Response of Feedback Amplifiers (Cont.)

- Poles s_1 & s_2 of $A_f(s)$



$$\frac{s}{\omega_0}=-\frac{1}{2Q}\pm\frac{1}{2Q}\sqrt{1-4Q^2}$$

$$s=-\frac{\omega_1+\omega_2}{2}\pm\frac{\omega_1+\omega_2}{2}\sqrt{1-4Q^2}$$

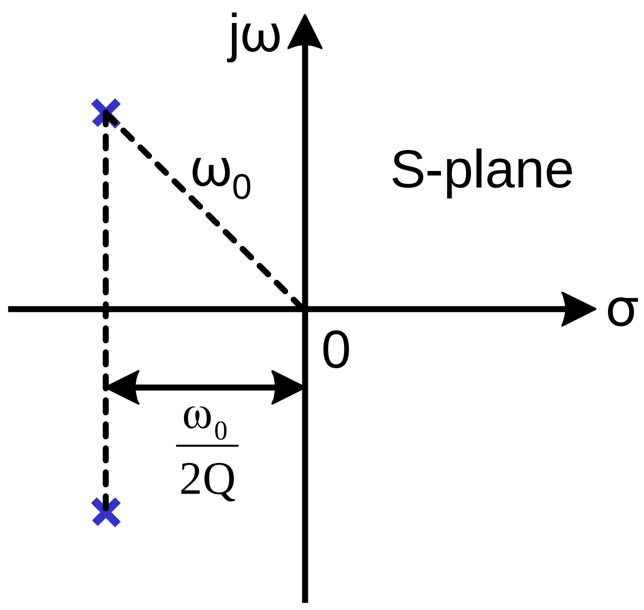
- For β=0

$$\beta A_0=0\Rightarrow\omega_0=\sqrt{\omega_1\omega_2}$$

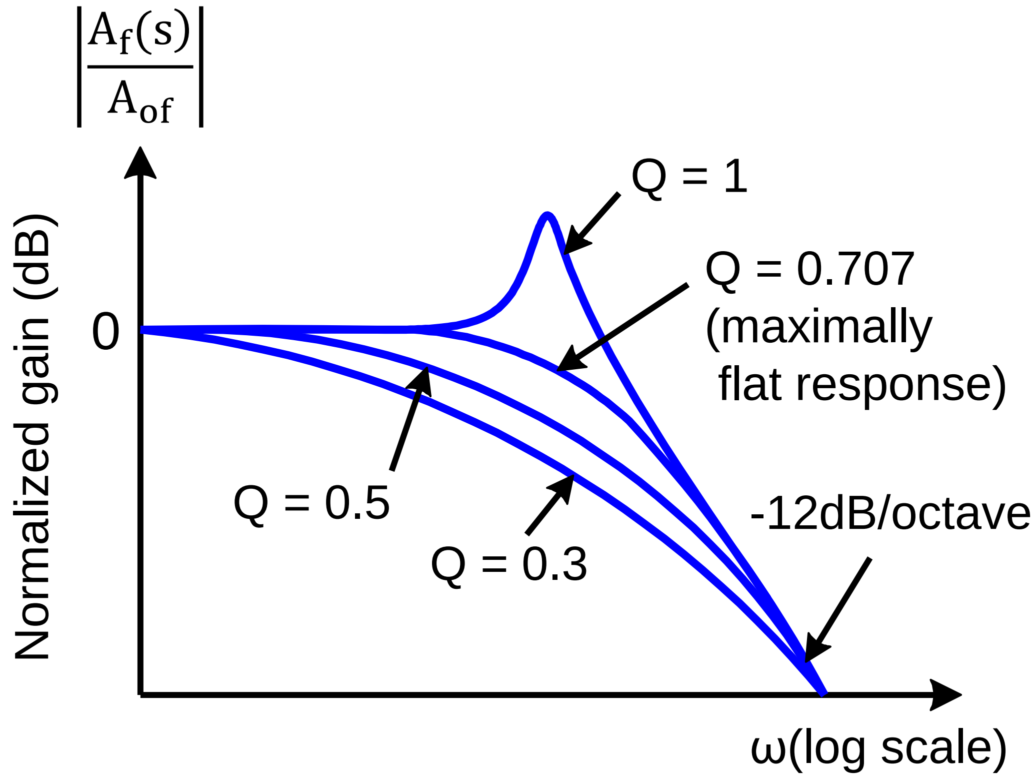
$$\Rightarrow Q_{min}=\frac{\sqrt{\omega_1\omega_2}}{\omega_1+\omega_2}\quad\text{when }\beta=0$$

Stability and Response of Feedback Amplifiers (Cont.)

◆ Definition of ω_0 and Q

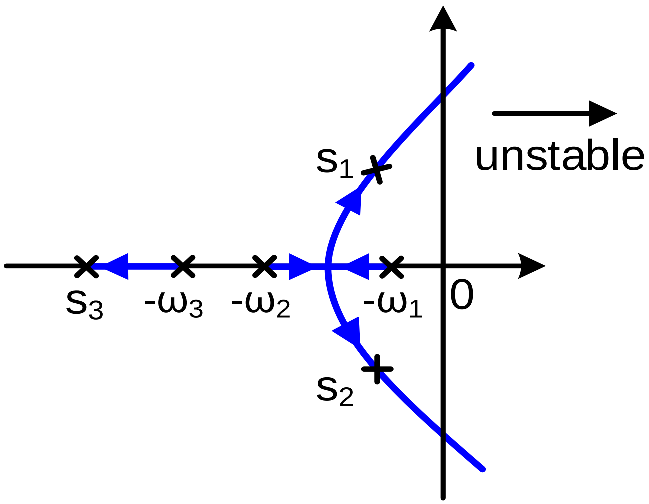


◆ Normalized gain ($20\log|\frac{A_f(s)}{A_{of}}|$) for various Q



Stability and Response of Feedback Amplifiers (Cont.)

- Three-pole function
  - $$A_f(s) = \frac{A_{Of}}{1 + \frac{s}{1+\beta A_O}\left(\frac{1}{\omega_1} + \frac{1}{\omega_2} + \frac{1}{\omega_3}\right) + \frac{s^2}{1+\beta A_O}\left(\frac{1}{\omega_1\omega_2} + \frac{1}{\omega_2\omega_3} + \frac{1}{\omega_1\omega_3}\right) + \frac{s^3}{(1+\beta A_O)\omega_1\omega_2\omega_3}}$$
    $$= \frac{A_{Of}}{1 + \frac{a_1 s}{1+\beta A_O} + \frac{a_2 s^2}{1+\beta A_O} + \frac{a_3 s^3}{1+\beta A_O}}$$
  - Root locus



Stability

- Stability definition
  - A system is stable if and only if all bounded input X(t) signals produce bounded output signals Y(t)
  - A signal Y(t) is bounded if $|Y(t)| \leq$ constant for all t

    For examples :

    1. left-plane pole results in output containing e^{pt}

       $p < 0 \Rightarrow$ bounded output
    2. Right-plane pole results in output containing e^{pt}

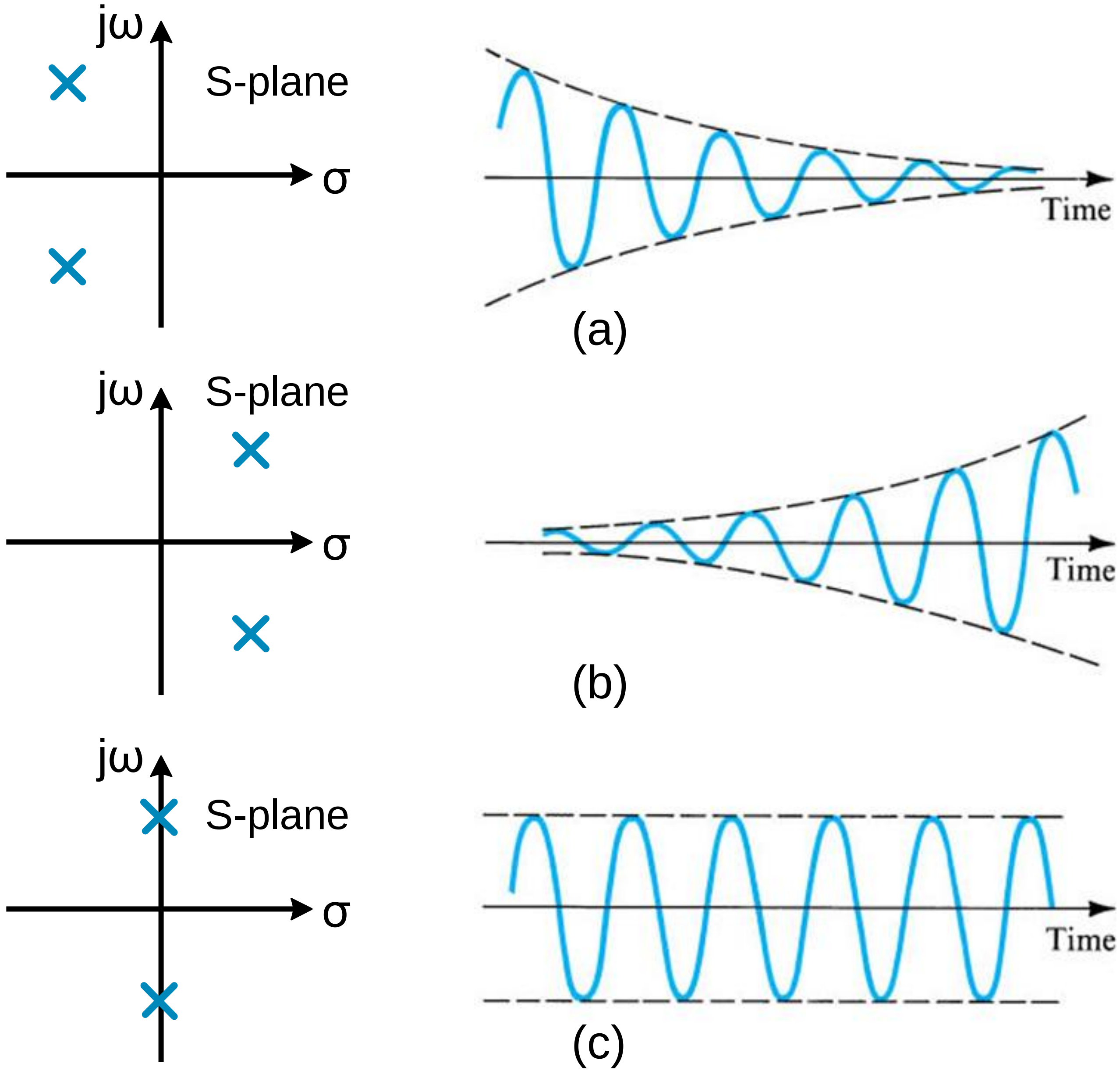
       $p > 0 \Rightarrow$ unbounded output
- Stability in feedback amplifiers

$$A_f(s) = \frac{A(s)}{1 + \beta A(s)}$$

  - The zeros of $(1+\beta A(s))$ are the poles of $A_f(s)$
  - If the amplifier without feedback is stable, all the poles of A(s) lie the left half plane.

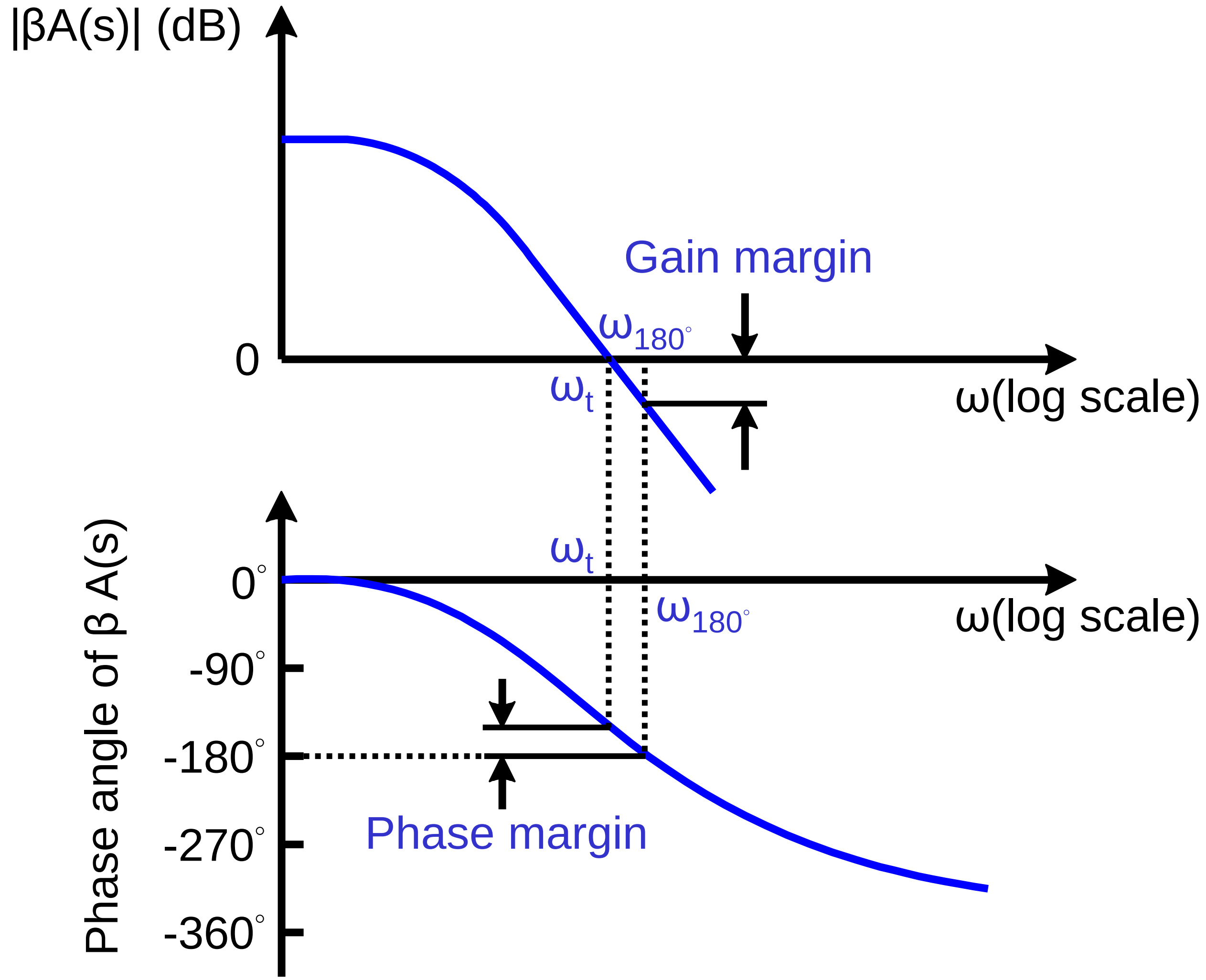
    If the feedback amplifier is stable, all the poles of $A_f(s)$, i.e. all of zeros of $(1+\beta A(s))$, lie in the left half plane.

Stability and Pole Location



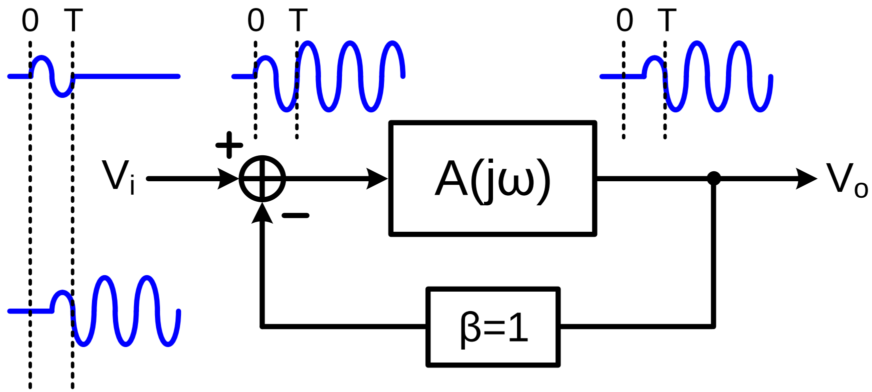
Stability Study Using Bode Plots

- Gain and phase margins

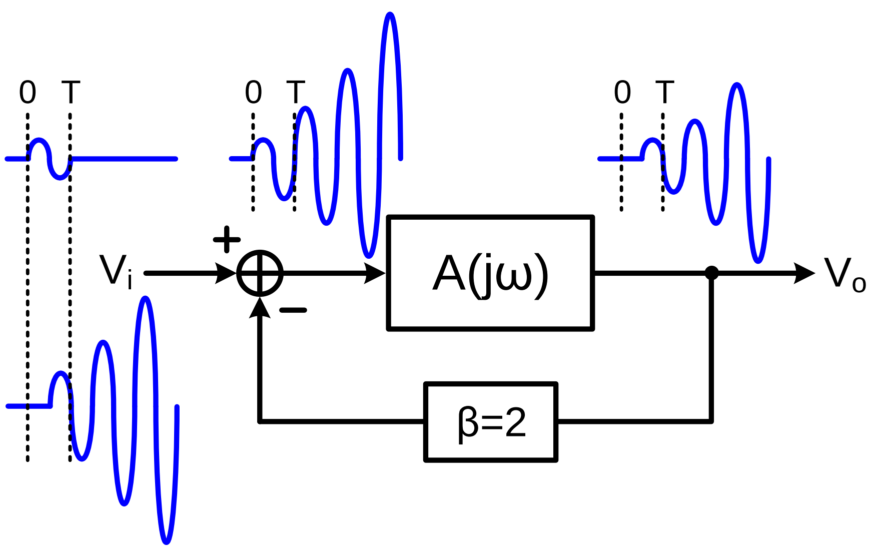


Stability vs. Loop Delay

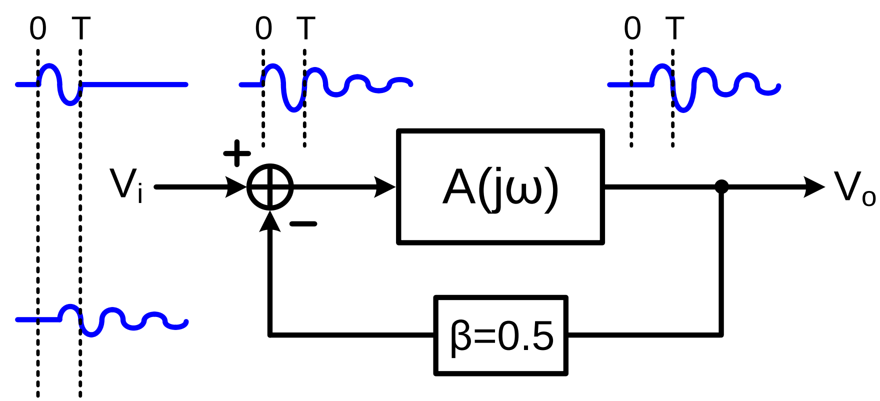
- Assume β is a constant, $|A(j\omega_0)| = 1$ and $\angle A(j\omega_0) = 180°$ → delay $\frac{T}{2}$, where T is period
  - $|\beta \cdot A(j\omega_0)| = 1$ for $\beta = 1$



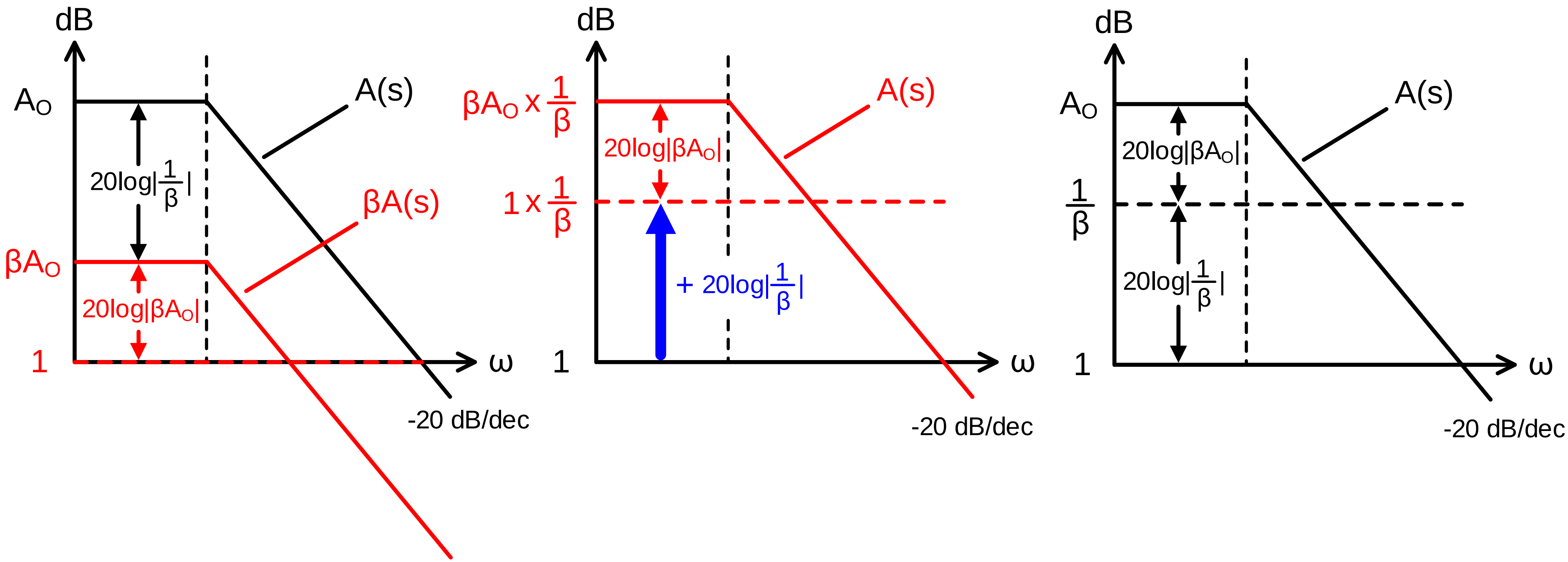
  - $|\beta \cdot A(j\omega_0)| = 2$ for $\beta = 2$



  - $|\beta \cdot A(j\omega_0)| = 0.5$ for $\beta = 0.5$



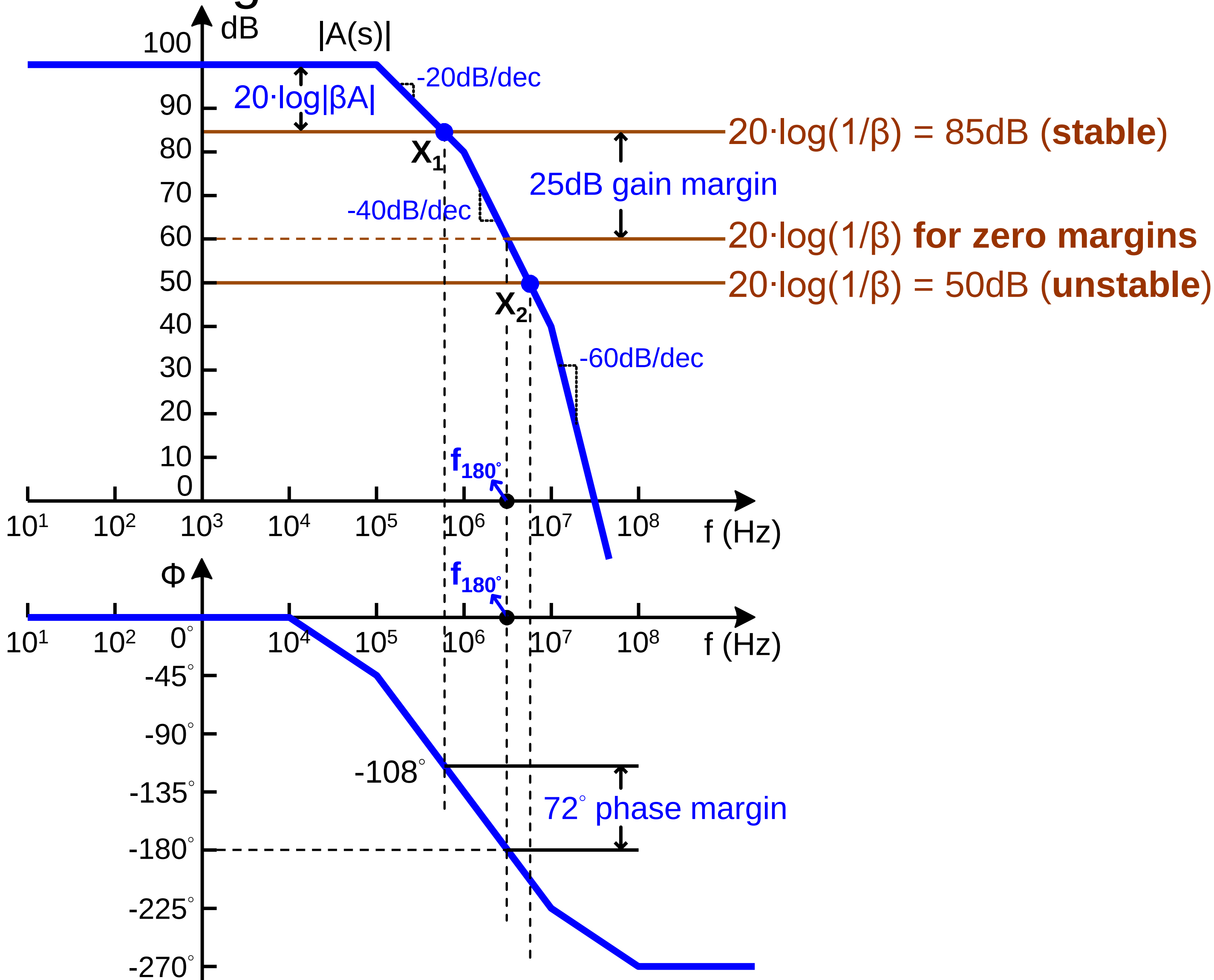
Bode Plot of Loop Gain



$$20\log|\beta A_O| + 20\log|\frac{1}{\beta}| = 20\log|\beta A_O \times \frac{1}{\beta}| = 20\log|A_O|$$

Stability Analysis Using Bode Plot of |A|

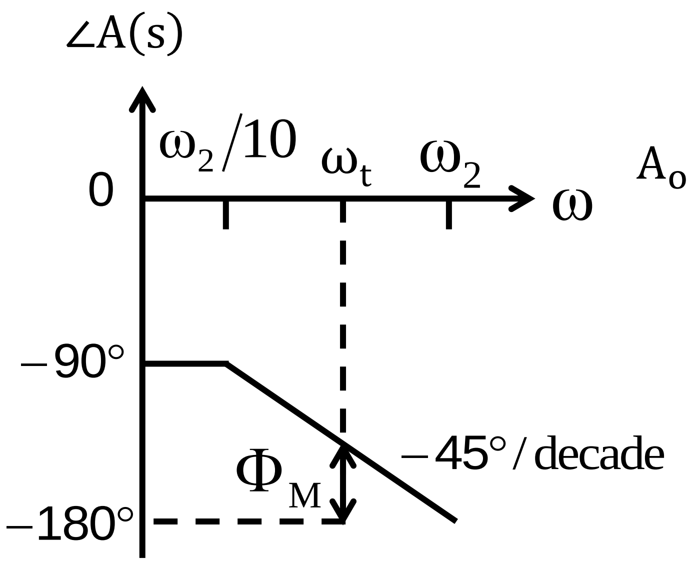
- Gain and phase margins

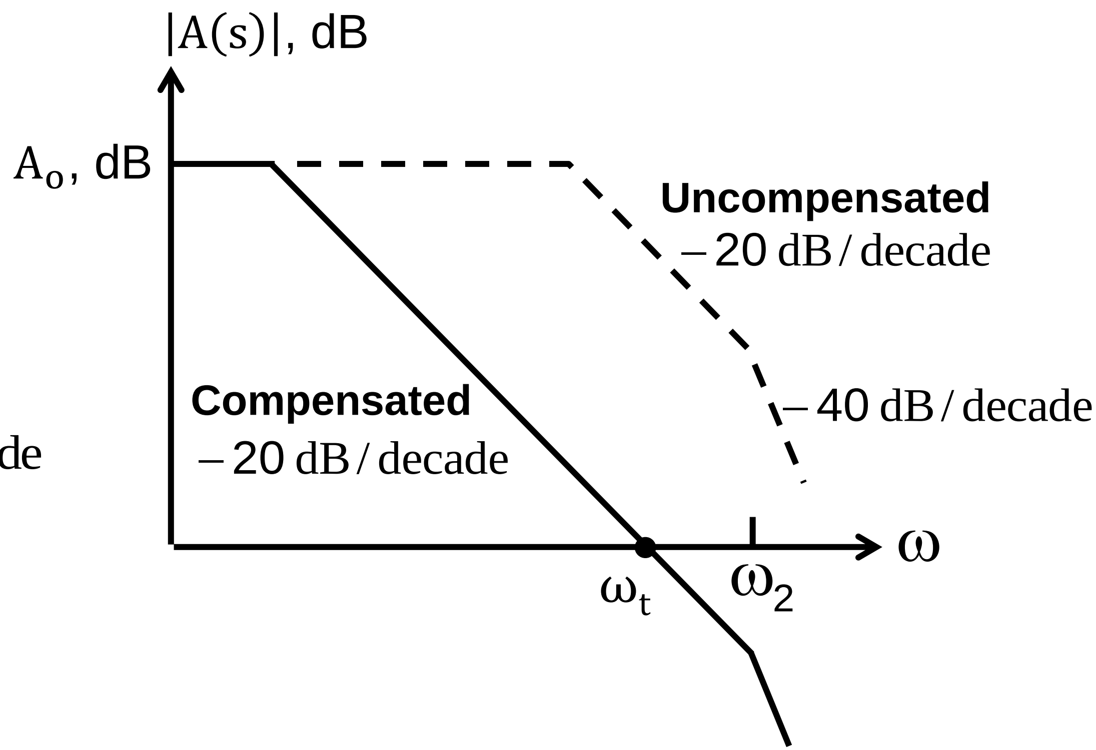


Compensation

- Additional components are inserted in the circuit, which alter the location of one or more poles of T(s) without changing T_O

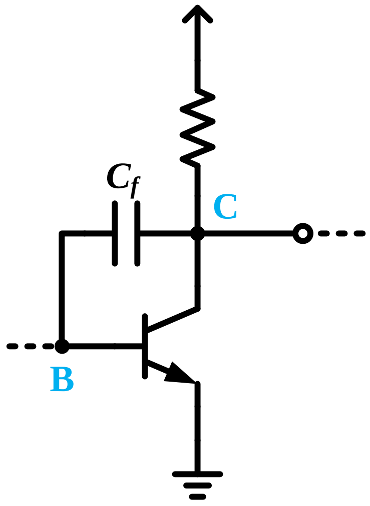
- Dominant-pole compensation



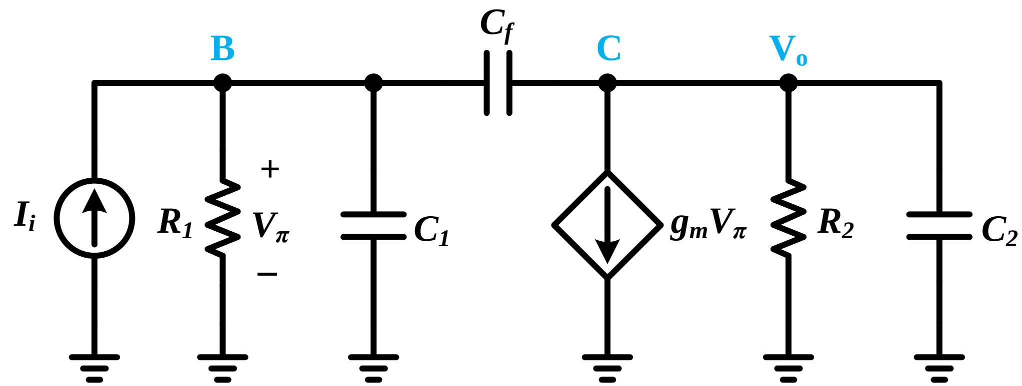


Miller Compensation and Pole Splitting

- A gain stage in a multistage amplifier with Miller compensation
- Small-signal equivalent circuit



(a)



(b)

Phase Margin of the Two-pole Feedback Amplifier

- Recall that

$$\omega_0 = \sqrt{\omega_1\omega_2(1+\beta A_O)} \ \& \ Q = \frac{\omega_0}{\omega_1+\omega_2}$$

$$s = -\frac{\omega_1+\omega_2}{2} \pm \frac{\omega_1+\omega_2}{2}\sqrt{1-4Q^2}$$

- ◆ Pole-separation factor $n = \dfrac{\omega_2}{\omega_1}$

$$\omega_0 = \omega_1\sqrt{n(1+\beta A_O)} \ \& \ Q = \frac{\sqrt{n(1+\beta A_O)}}{n+1}$$

$$s = -\frac{\omega_1(1+n)}{2}(1\pm\sqrt{1-4Q^2})$$

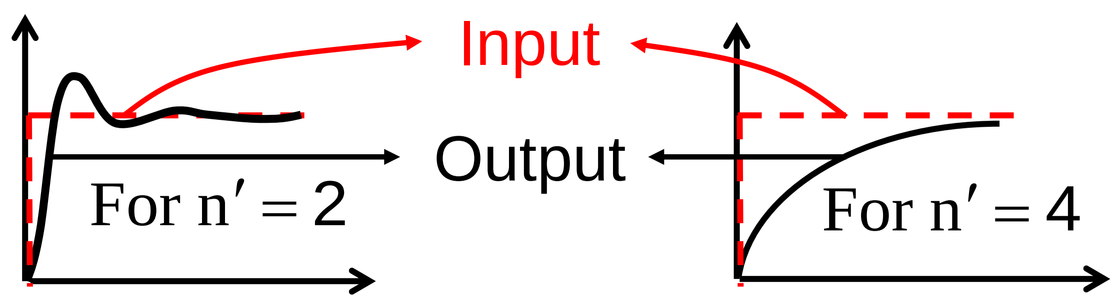
Phase Margin of the Two-pole Feedback Amplifier (Cont.)

If $n >> 1 \quad \Rightarrow \quad n \approx \frac{1+\beta A_O}{Q^2} \approx \frac{\beta A_O}{Q^2}$ (if $\beta A_O >> 1$)

Let $n' = \frac{n}{1+\beta A_o} = \frac{1}{Q^2}$, then $\frac{\omega_2}{\omega_1} = n \Rightarrow \frac{\omega_2}{\omega_t} = n'$ $\because \omega_t \fallingdotseq \omega_1 \beta A_o$ (gain-bandwidth product)

$$\begin{cases} Q = \frac{1}{\sqrt{2}} \Rightarrow n' = 2 \text{ (fast response)} & \text{Phase margin}=63^\circ \\ Q = \frac{1}{\sqrt{3}} \Rightarrow n' = 3 & \text{Phase margin}=71^\circ \\ Q = \frac{1}{2} \Rightarrow n' = 4 \text{ (critically damped)} & \text{Phase margin}=76^\circ \end{cases}$$

Step response



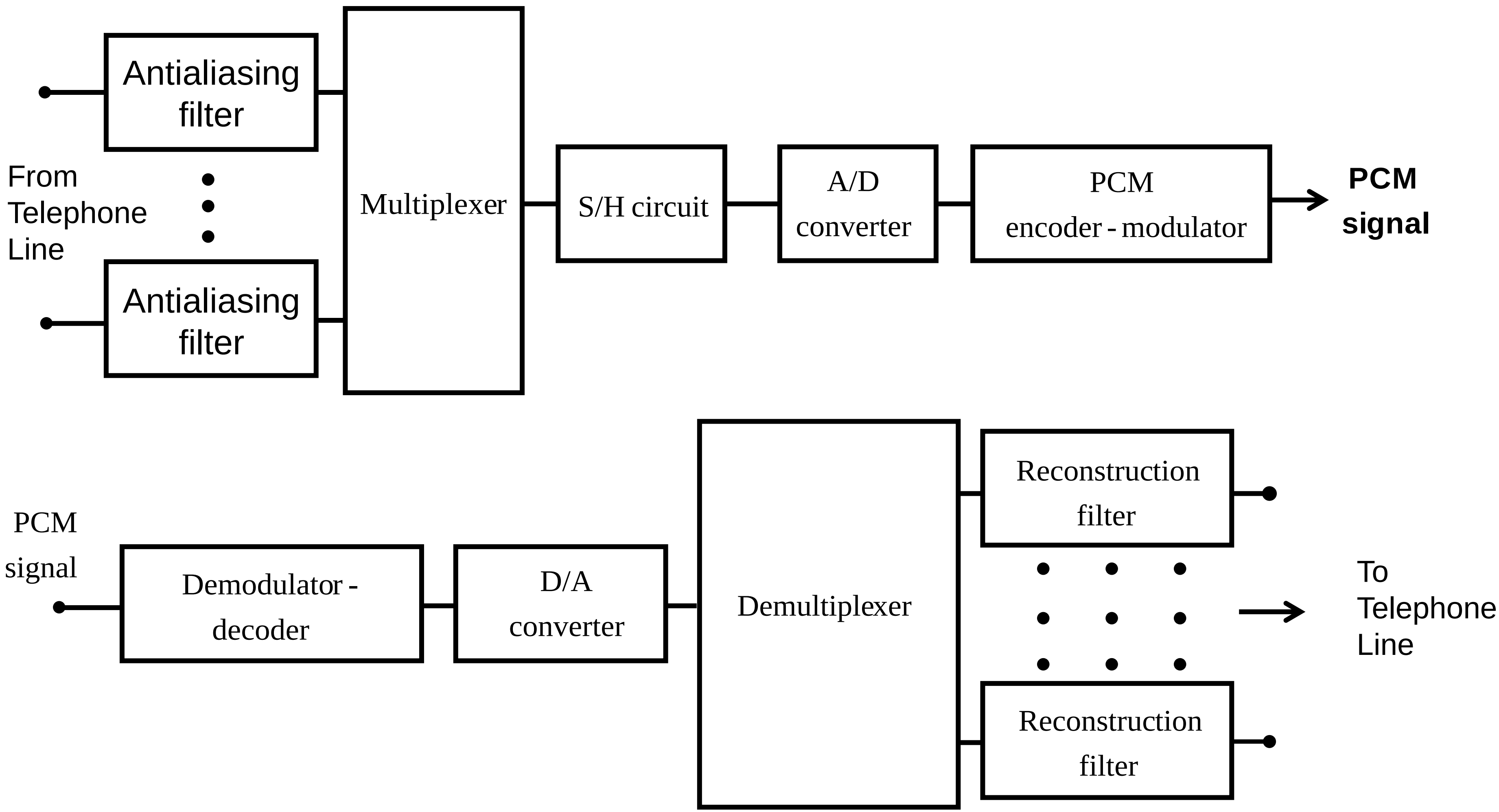
★ $n'=2$ is usually an optimal choice for fast response when ω_2 is fixed

Appendix(I)

- Signal processing system
- Digital-to-Analog converter (DAC)
- R-2R ladder-type DAC
- Current-Mode DAC
- Analog-to-Digital converter (ADC)

Signal Processing System

- Example：Telephone system



Signal Processing System (Cont.)

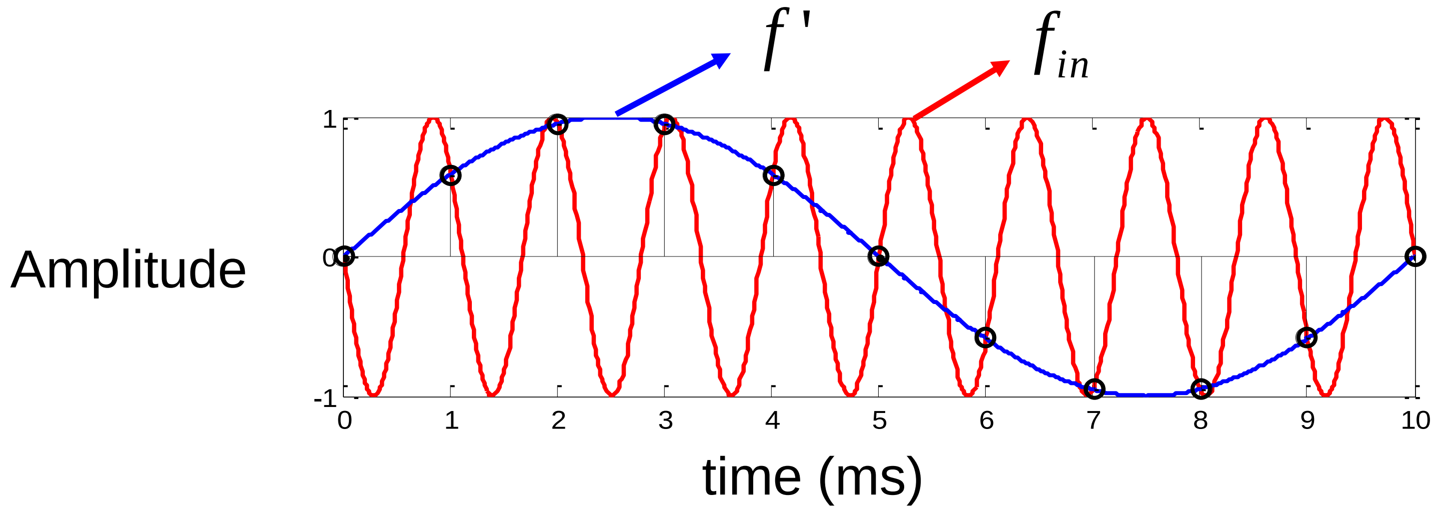
- Illustration of aliasing

$$f_s \textbf{ (Sampling frequency)} = 1k\ Hz$$

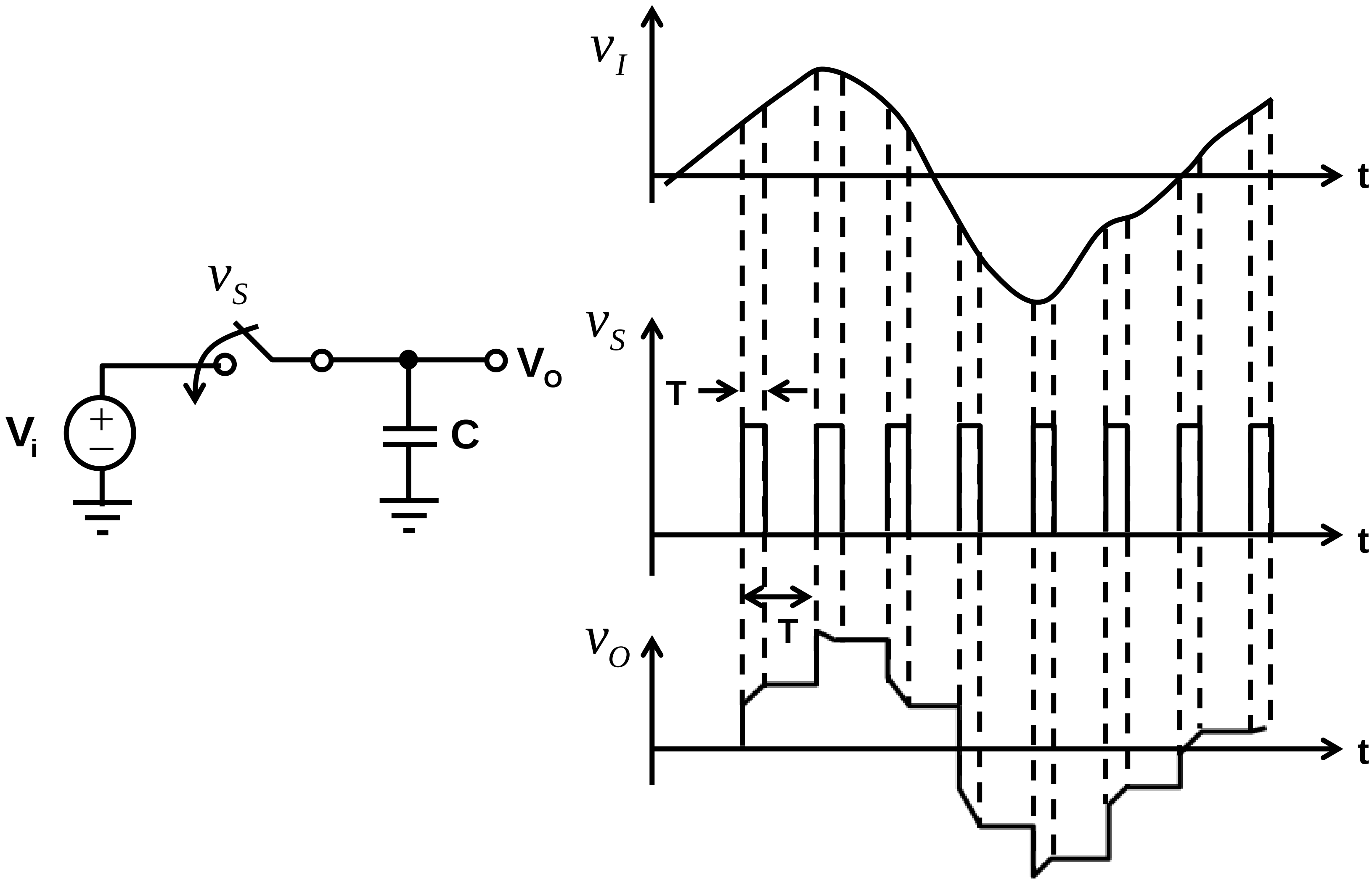
$$f_{in} \textbf{ (Input frequency)} = 900\ Hz$$

$$f_{in} \geq \frac{1}{2} f_s \Rightarrow \textbf{Aliasing}\ f' = \left| f_{in} - f_s \right| = 100\ Hz$$

Nyquist theorem: $f_s > 2f_{in}$ to avoid aliasing effect



Sample and Hold System



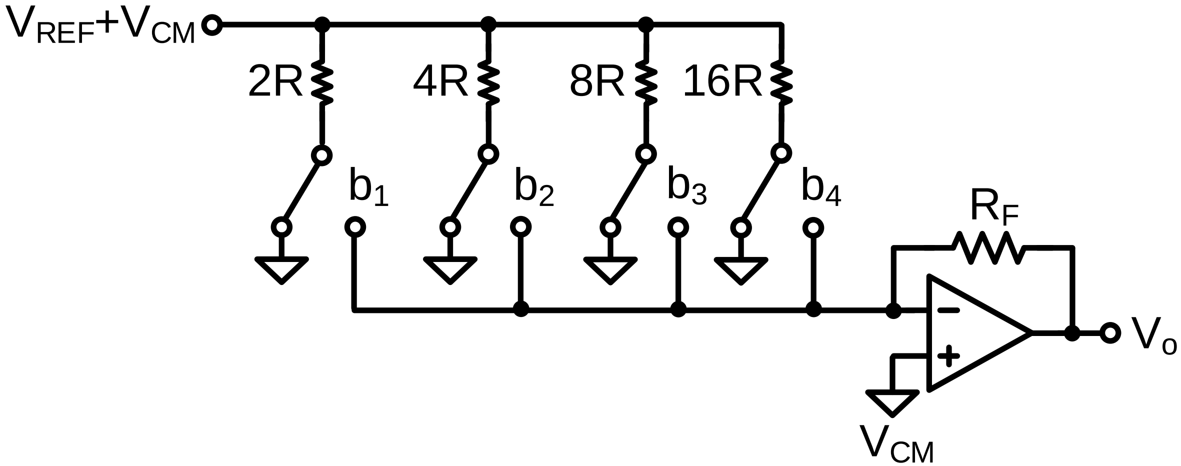
Digital-to-Analog Converter (DAC)

- Digital input, $b_1, b_2, \ldots, b_N$; Analog output, V_o

  For a N-bit DAC, $V_o = (2^{N-1} b_1 + 2^{N-2} b_2 + \ldots + 2^1 b_{N-1} + b_N) V_{LSB}$

  where $V_{LSB} = \dfrac{R_F}{2^N R} V_{REF}$ (LSB: least significant bit)

- 4-bit resistor DAC example

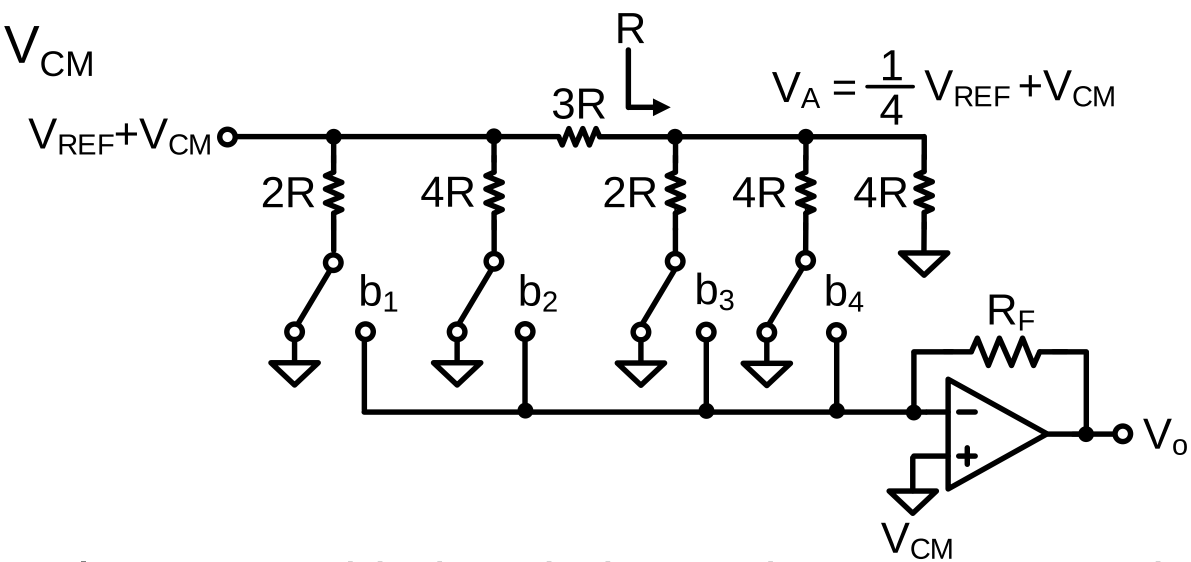


$$V_o = -R_F V_{REF}\left(\frac{b_1}{2R} + \frac{b_2}{4R} + \frac{b_3}{8R} + \frac{b_4}{16R}\right) + V_{CM} = \left(\frac{-R_F}{R} V_{REF}\right) b_{in} + V_{CM}$$

where $b_{in} = b_1 2^{-1} + b_2 2^{-2} + b_3 2^{-3} + b_4 2^{-4}$

Reduced-Resistance-Ratio Ladders

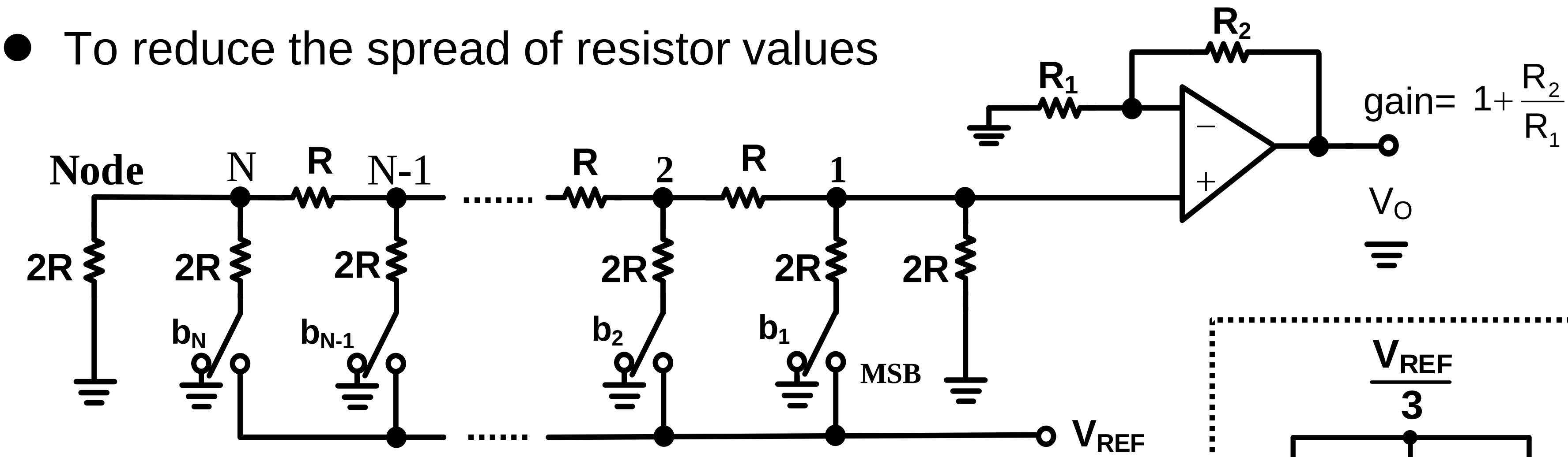
- Reduce the large resistor ratios in a binary-weighted array
- Introduce a series resistor to scale signals in portions of the array V_A= 1/4 V_{ref} + V_{CM}



- An additional 4R was added such that resistance seen to the right of the 3R equals R.
- One-fourth the resistance ratio compared to the binary-weighted case
- Current ratio has remained unchanged
  - ◆ Switches must be scaled in size
- Repeating this procedure recursively, one can obtain an R-2R ladder

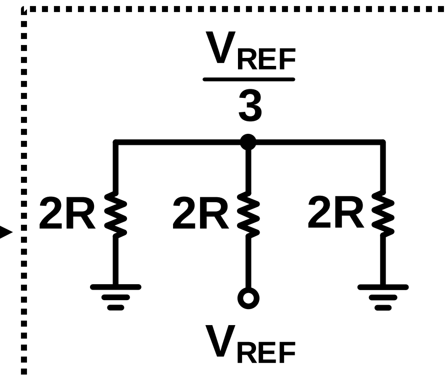
R-2R Ladder-Type DAC

- To reduce the spread of resistor values



- MSB is 1 & all other bits are 0
  → V_1=V_{REF}/3, $V_o = {}^{V_{REF}}/_{3} \cdot (1 + {}^{R_2}/_{R_1})$

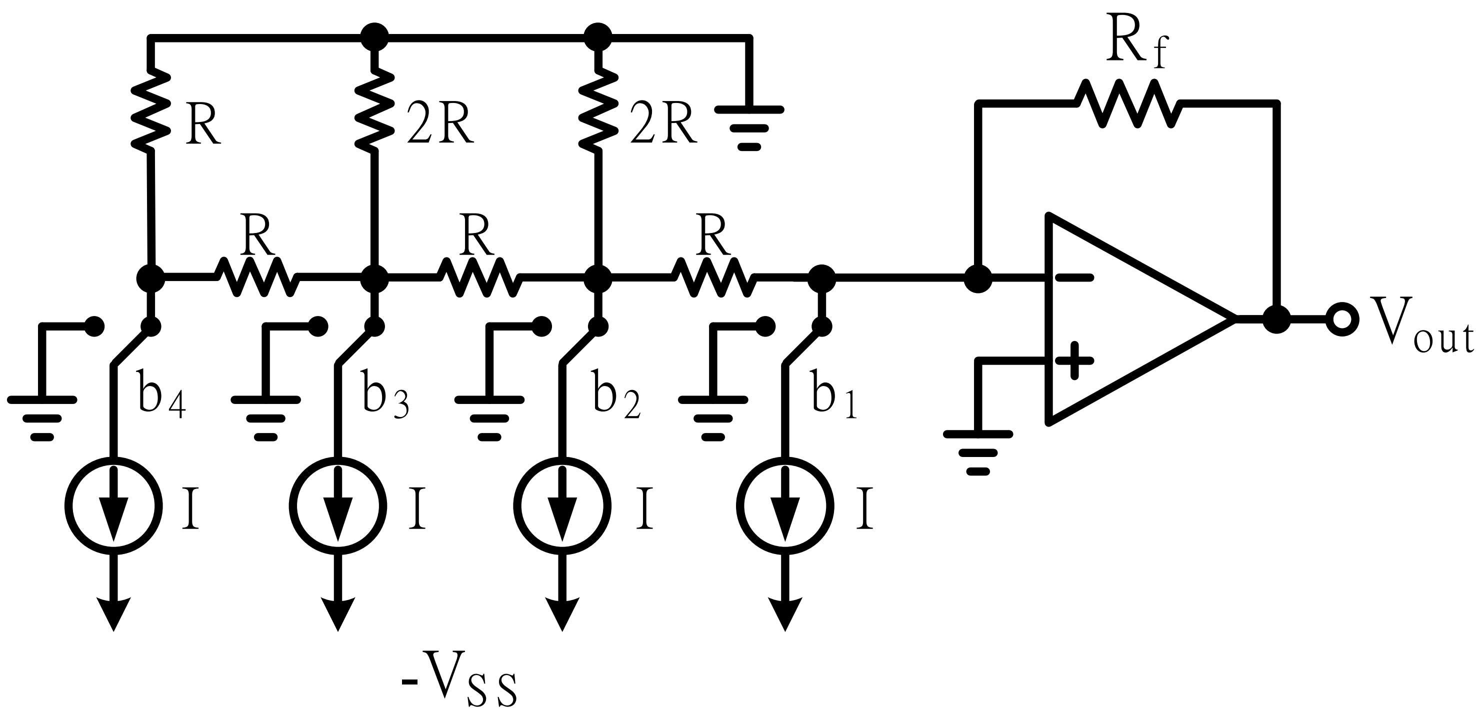
Equivalent circuit



- Bit 2 is 1 & all other bits are 0
  → V_2=V_{REF}/3 & V_1=V_{REF}/6, $V_o = {}^{1}/_{2} \cdot {}^{V_{REF}}/_{3} \cdot (1 + {}^{R_2}/_{R_1})$
- Bit 3 is 1 & all other bits are 0
  → V_3=V_{REF}/3 & V_2=V_{REF}/6 & V_1=V_{REF}/12, $V_o = {}^{1}/_{4} \cdot {}^{V_{REF}}/_{3} \cdot (1 + {}^{R_2}/_{R_1})$
- Output resistance at nodes from 1 to N are all the same, i.e. $({}^{2}/_{3})R$
- Multiplying DAC
  → Use varying analog signal V_a instead of fixed reference voltage V_{REF}

R-2R Ladder-Type DAC (Cont.)

- R-2R ladder DAC with equal currents through switches
  - Slower since the internal nodes exhibit some voltage swings(as opposed to the previous configuration where internal nodes all remain at fixed voltage)



R-2R Ladder-Type DAC (Cont.)

- Assume only $b_1 = 1$, the current drawn through R_f is I

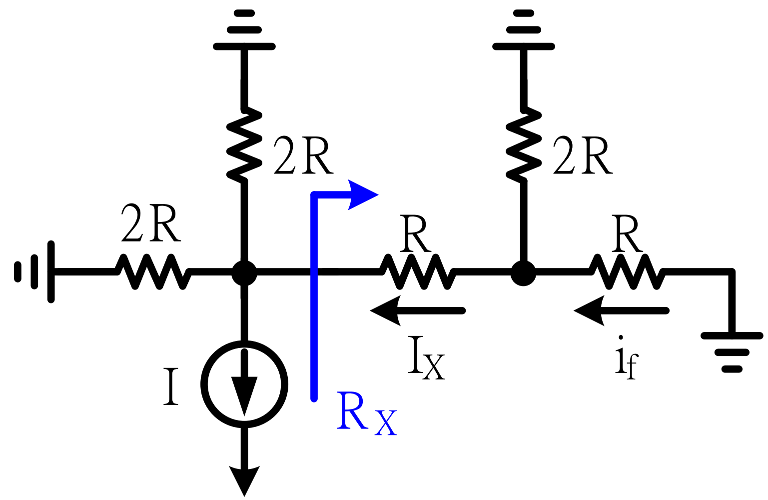
  Assume only $b_2 = 1$, the current drawn through R_f is I/2

  Assume only $b_3 = 1$, the equivalent circuitto find the current through R_f is:

$$R_X = R + \frac{2}{3}R = \frac{5}{3}R$$

$$I_X = I \times \frac{(2R||2R)}{(2R||2R) + \frac{5}{3}R} = \frac{3}{8}I$$

$$i_f = \frac{2R}{2R+R} I_X = \frac{1}{4}I$$



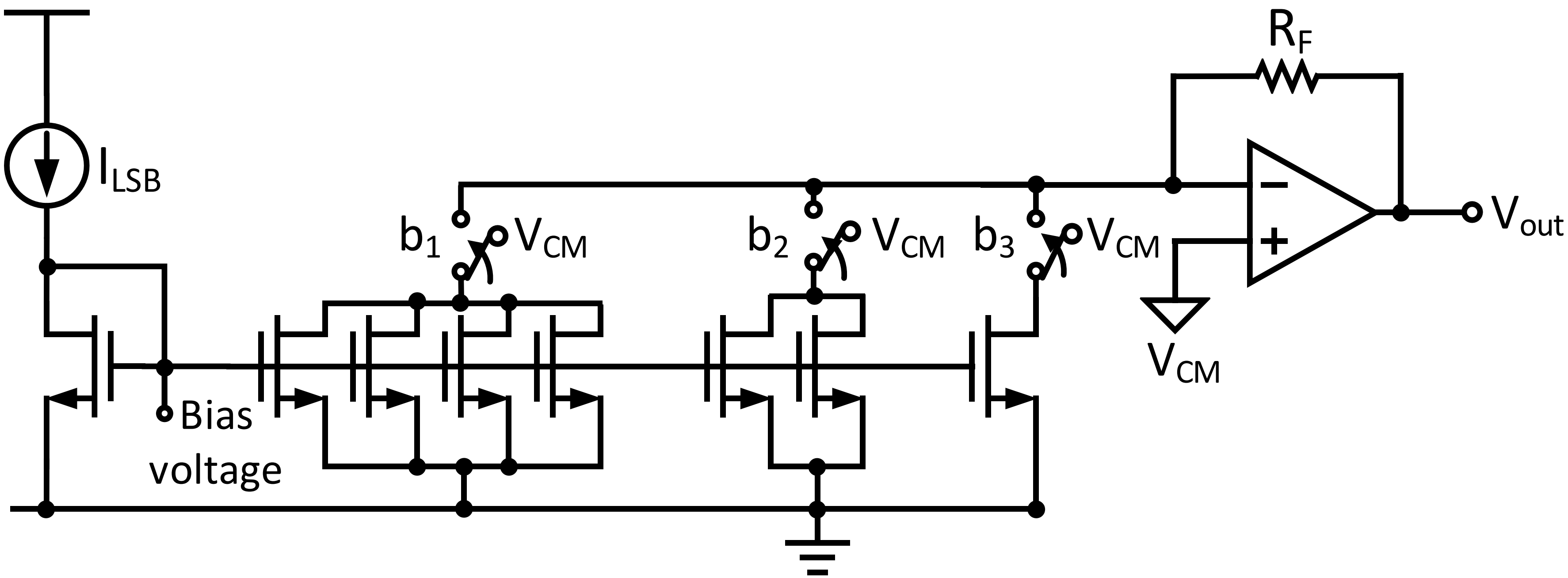
- With similar analysis, it can be shown that the current through b_4&R_f is I/8. Therefore,

$$V_{out} = 2R_f \times I(2^{-1}b_1 + 2^{-2}b_2 + 2^{-3}b_3 + 2^{-4}b_4)$$

Current-Mode DAC

- High-speed
- Switch current to output or to V_{CM}

  The output current is converted to a voltage through R_F
- The upper portion of current source always remains at V_{CM} potential.



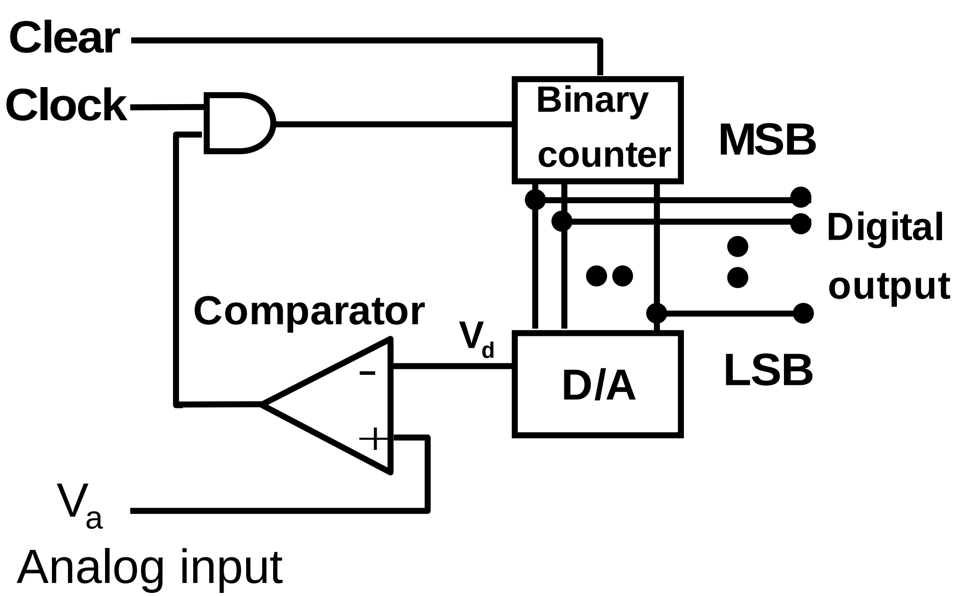
$$V_{out} = (2^2 b_1 + 2^1 b_2 + 2^0 b_3) \cdot I_{LSB} R_F + V_{CM}$$

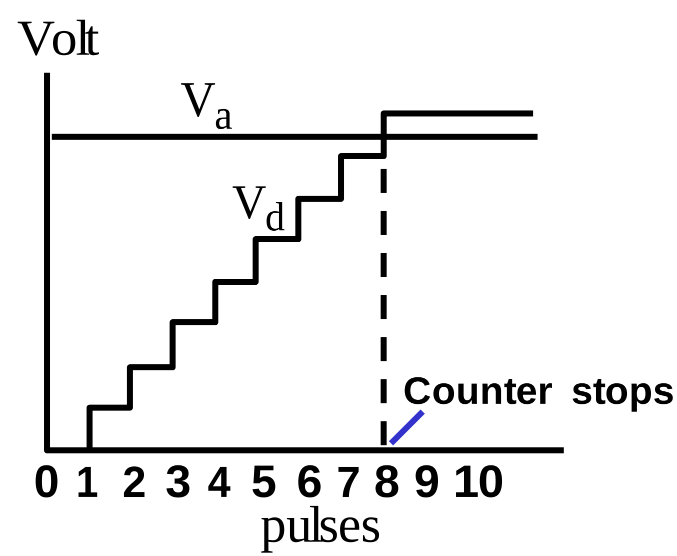
Analog-to-Digital Converters (ADC)

1. Counting ADC
2. Dual-slope (ratiometric) ADC
3. Flash (parallel-comparator) ADC
4. Successive-approximation ADC

Speed : 3>4>1>2

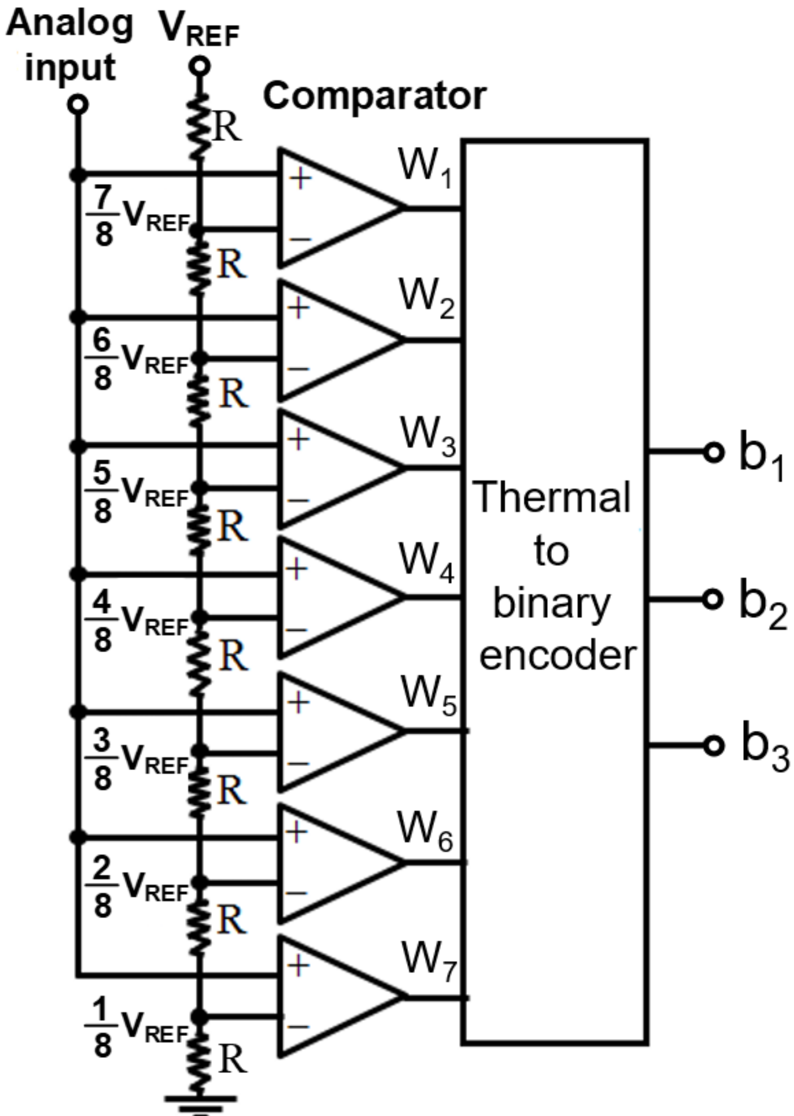
- Counting ADC





Flash (Parallel-Comparator) ADC

- Block diagram

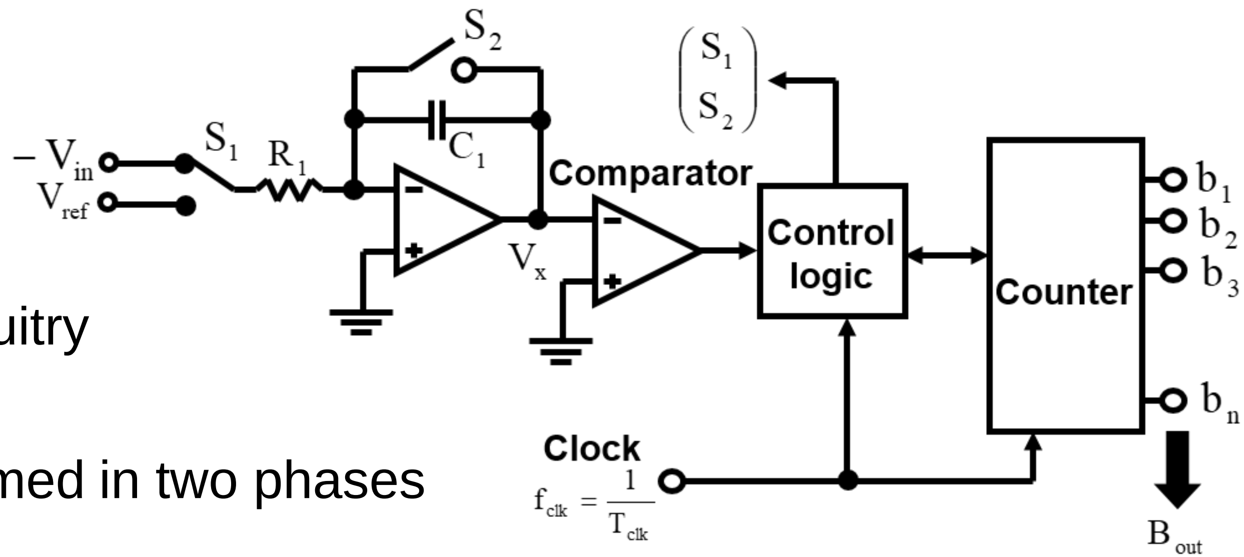


- Truth table of encoder

Inputs							Outputs		
Thermometer code							Binary code		
W_1	W_2	W_3	W_4	W_5	W_6	W_7	b_1	b_2	b_3
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	1
0	0	0	0	0	1	1	0	1	0
0	0	0	0	1	1	1	0	1	1
0	0	0	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	1
0	1	1	1	1	1	1	1	1	0
1	1	1	1	1	1	1	1	1	1

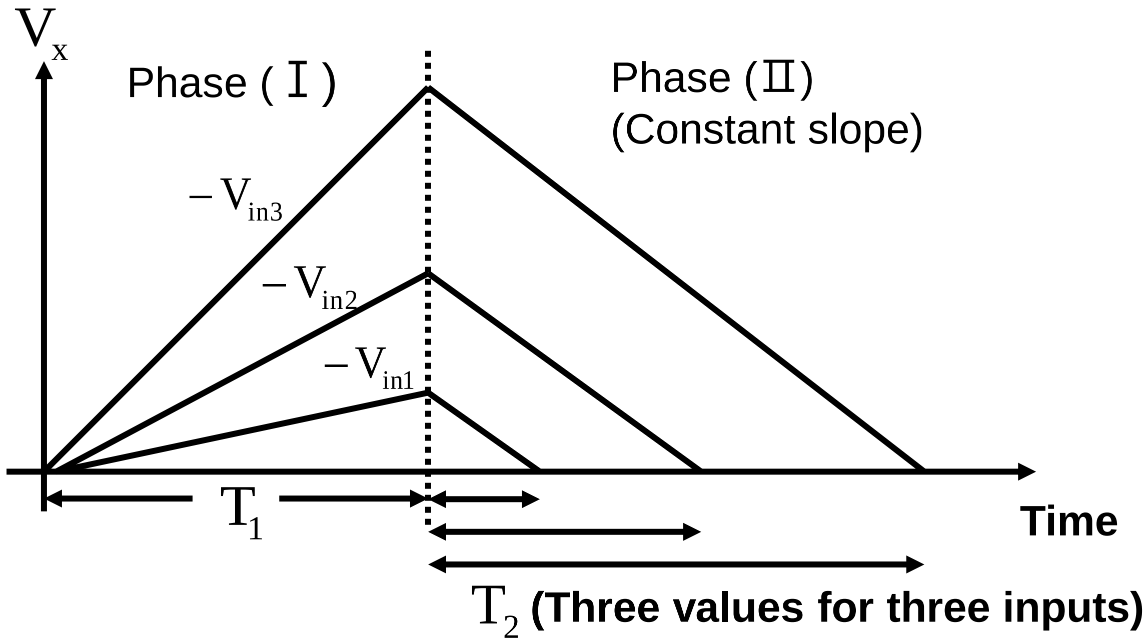
Dual-Slope (Ratiometric) ADC

- A popular approach for realizing high-accuracy data conversion on very slow-moving signals
- Very low offset error
  Very low gain error
  Highly linear
  Small amount of circuitry



- Conversion is performed in two phases
- Phase I
  - It's a fixed time interval of length T_1
    $T_1=2^N T_{clk}$ where T_{clk} is the period for one clock cycle
  - S_1 is connected to $-V_{in}$ such that V_x ramps up proportional to the magnitude of V_{in}
  - At the beginning, V_x is reset to zero by S_2
  - At the end of phase I, $V_x(T_1) = \int_0^{T_1} \frac{V_{in}}{R_1}\frac{1}{C_1}dt = \frac{V_{in}T_1}{R_1C_1}$

Dual-Slope (Ratiometric) ADC (Cont.)



- Phase Ⅱ
  - A variable amount of time, T_2
  - At the beginning, counter is reset and S_1 is connected to V_{ref} , resulting in a constant slope for the decaying voltage at V_x
  - The counter simply counts until V_x is less than zero

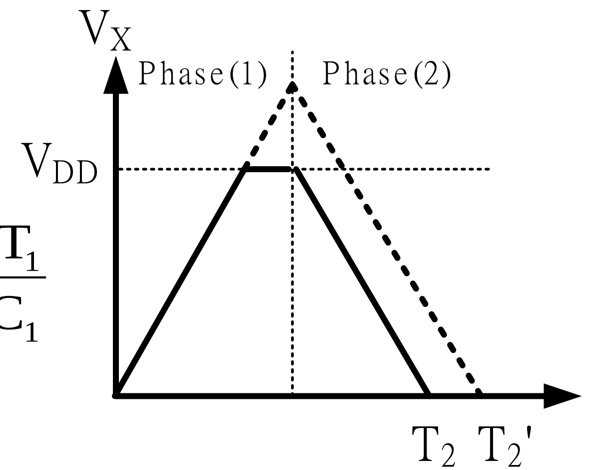
Dual-Slope (Ratiometric) ADC (Cont.)

- Assuming the digital output count is normalized so that the largest count is unity, the counter output B_{out}, can be defined to be

  $B_{out} = b_1 2^{-1} + b_2 2^{-2} + \ldots + b_N 2^{-N}$

  and we have

  $T_2 = 2^N B_{out} T_{clk} = (b_1 2^{N-1} + b_2 2^{N-2} + \ldots + b_N)\, T_{clk}$



$$V_x(t) = -\int_{T_1}^{t} \frac{V_{ref}}{R_1 C_1} d\tau + V_x(T_1) = -\frac{V_{ref}}{R_1 C_1}(t - T_1) + \frac{V_{in} T_1}{R_1 C_1}$$

Since $V_x(t) = 0$, when $t = T_1 + T_2$

$$\frac{-V_{ref} T_2}{R_1 C_1} + \frac{V_{in} T_1}{R_1 C_1} = 0 \quad \Rightarrow T_2 = T_1\left(\frac{V_{in}}{V_{ref}}\right)$$

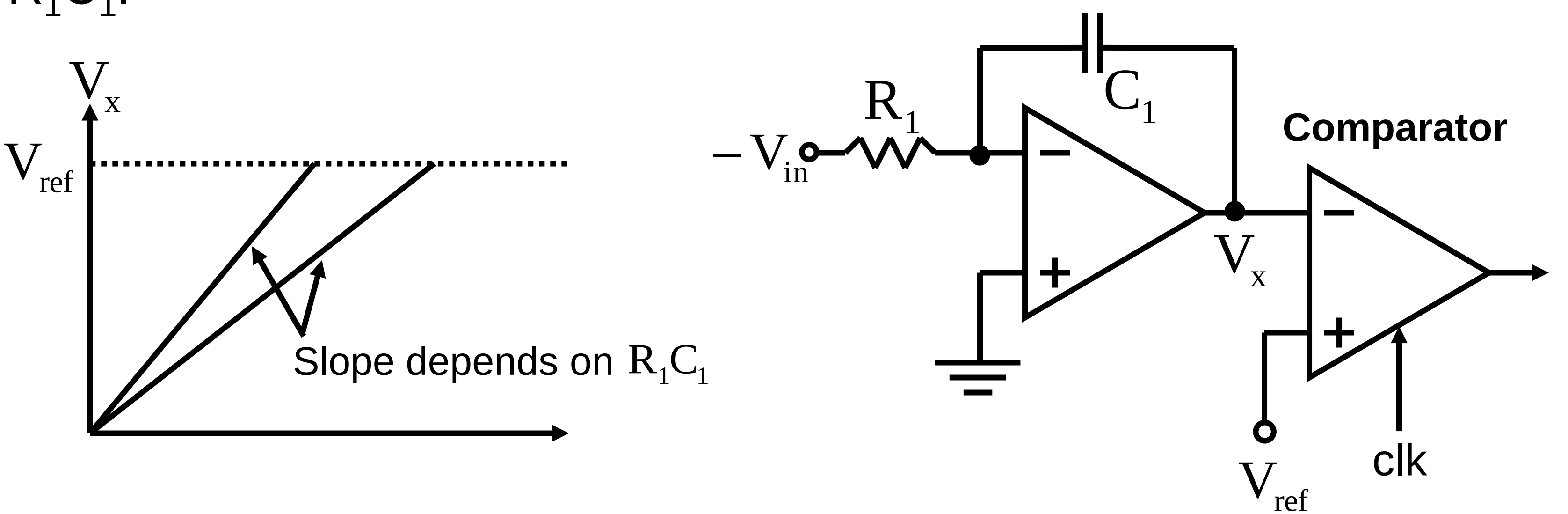
$$\Rightarrow B_{out} = b_1 2^{-1} + b_2 2^{-2} + \cdots + b_N = \frac{V_{in}}{V_{ref}} = \frac{T_2}{T_1}$$

- From the above equations, the digital output does not depend on the time constant, $R_1 C_1$. R_1 and C_1 should be chosen such that a reasonable large peak value of V_x is obtained without clipping to reduce noise effects.

Reading Assignment: Dual-Slope (Ratiometric) ADC (Cont.)

- From the above equations, the digital output does not depend on the time constant, R_1C_1.

  R_1 and C_1 should be chosen such that a reasonable large peak value of V_x is obtained without clipping to reduce noise effects.
- For a single-slope conversion, gain error occurs and is a function of R_1C_1.



- To increase resolution and speed, multi-slope conversion can be used.

Reading Assignment: Dual-Slope (Ratiometric) ADC (Cont.)

- Offset error and gain error can be calibrated
  (very important mostly in DC measurement)
  - Measure zero input first , then memorize its digital output, B_x
  - Measure full-scale DC signal, then memorize its digital output, B_y
    Gain error = $(B_y - B_x) - (2^N - 1)$
  - Final calibrated output $B_{out} = (B_{out} - B_x) \times \dfrac{2^N - 1}{B_y - B_x}$
- Quite slow
  $2 \cdot 2^N$ clocks are required (worse case), e.g. for a 16-bit converter with a clock frequency equal to1MHz, the worst-case conversion time is around 7.6Hz.

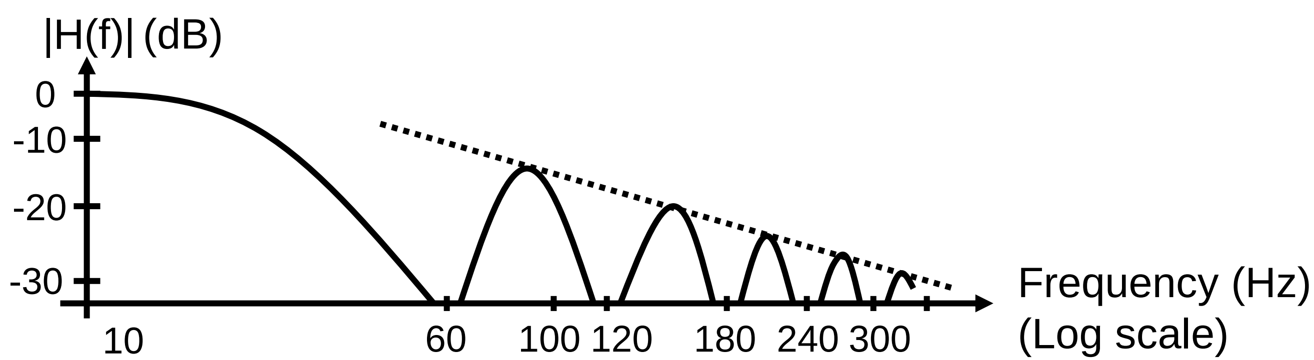
Reading Assignment: Dual-Slope (Ratiometric) ADC (Cont.)

- Effective input filter with sinc function
  - By a careful choice for T_1, certain frequency components superimposed on the input signal can be significantly attenuated
  - If $V_{in}(t) = V_{in}\cos(2\pi ft + \theta)$, where V_{in} are arbitrary magnitude

$$V_x(T_1) = \int_0^{T_1} \frac{V_{in}\cos(2\pi ft)}{R_1C_1} dt = \frac{V_{in}\sin(2\pi ft)}{2\pi fR_1C_1}\bigg|_0^{T_1} = \frac{V_{in}T_1}{R_1C_1}\frac{\sin(2\pi fT_1)}{2\pi fT_1}$$

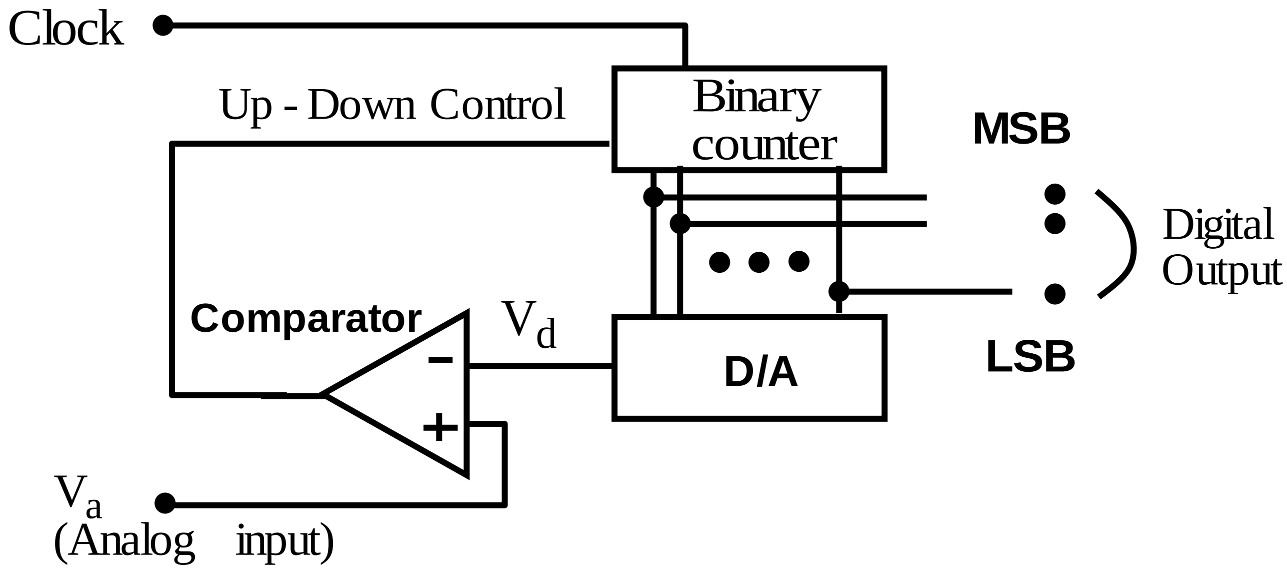
$$V_x(T_1) \approx H(f) \times \frac{V_{in}T_1}{R_1C_1} \text{ when } fT_1 \text{ approaches } 1, 2, 3, \ldots$$

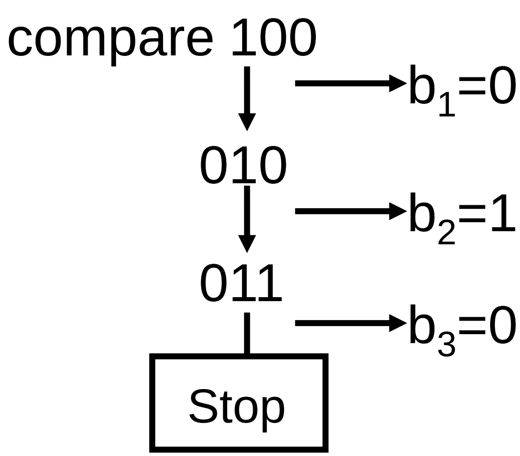
  - Filter transfer function $|H(f)| = \left|\frac{\sin(\pi fT_1)}{\pi fT_1}\right| = |\mathrm{sinc}(\pi fT_1)|$
- An example to filter out power line noise, especially 60Hz
  - T_1 is chosen to be an integer multiples of 16.67ms.
  - 60Hz, 120Hz, 180Hz, …… are suppressed.

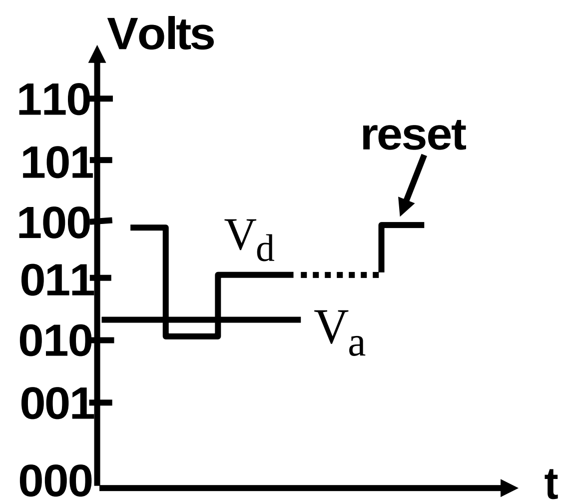


Successive-Approximation (SA) ADC

- Basic of successive-approximation ADC



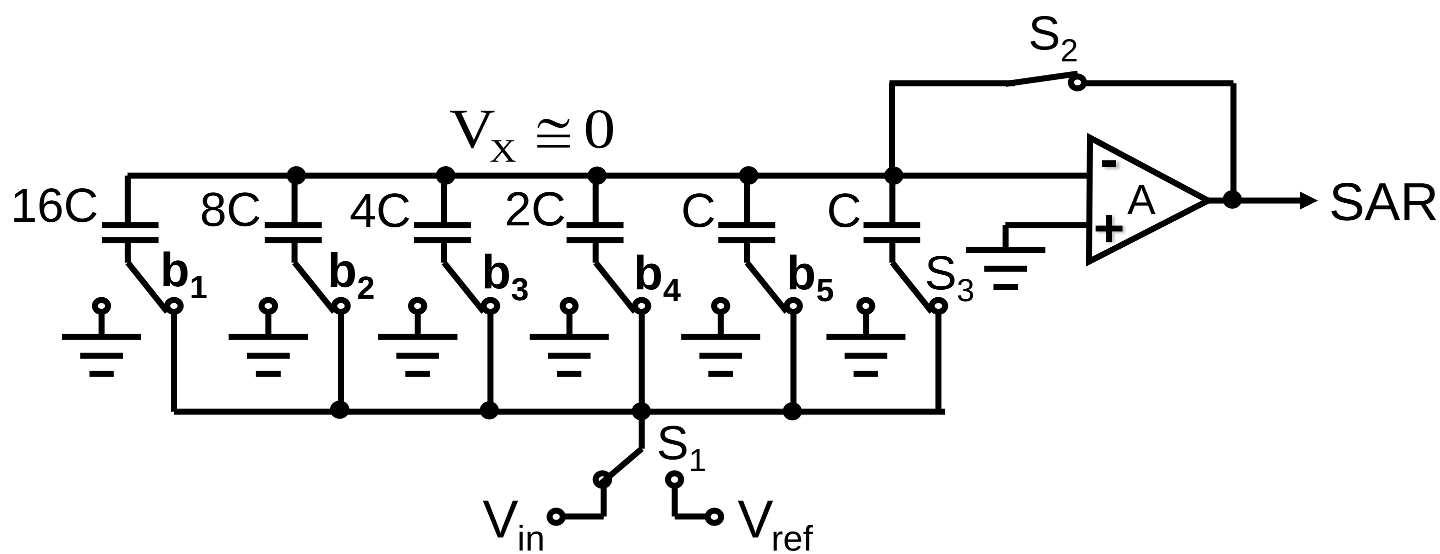




Charge-Redistribution Successive-Approximation (SA) ADC

- Example : A 5-bit ADC
  - 3 operational modes
    - Sample mode

      Comparator is reset though S_2. All capacitors are charged to V_{in}, which performs S/H



Charge-Redistribution SA ADC (Cont.)

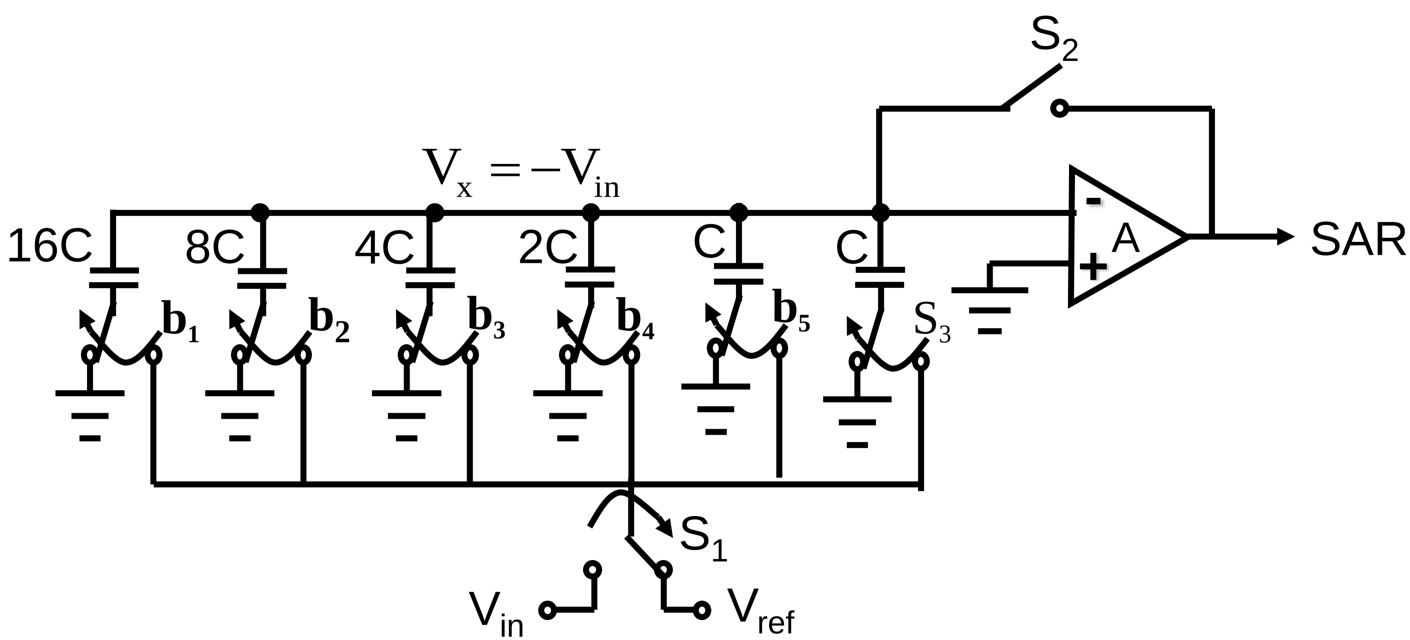
- Hold mode

  Comparator is taken out of reset.

  All capacitors are switched to ground.

  V_x : $-V_{in}$

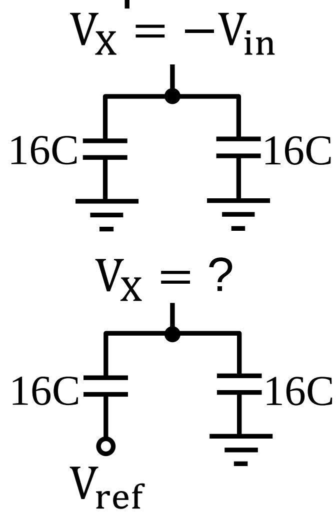
  V_{in} is held on the capacitor array



Charge-Redistribution SA ADC (Cont.)

- Bit cycling

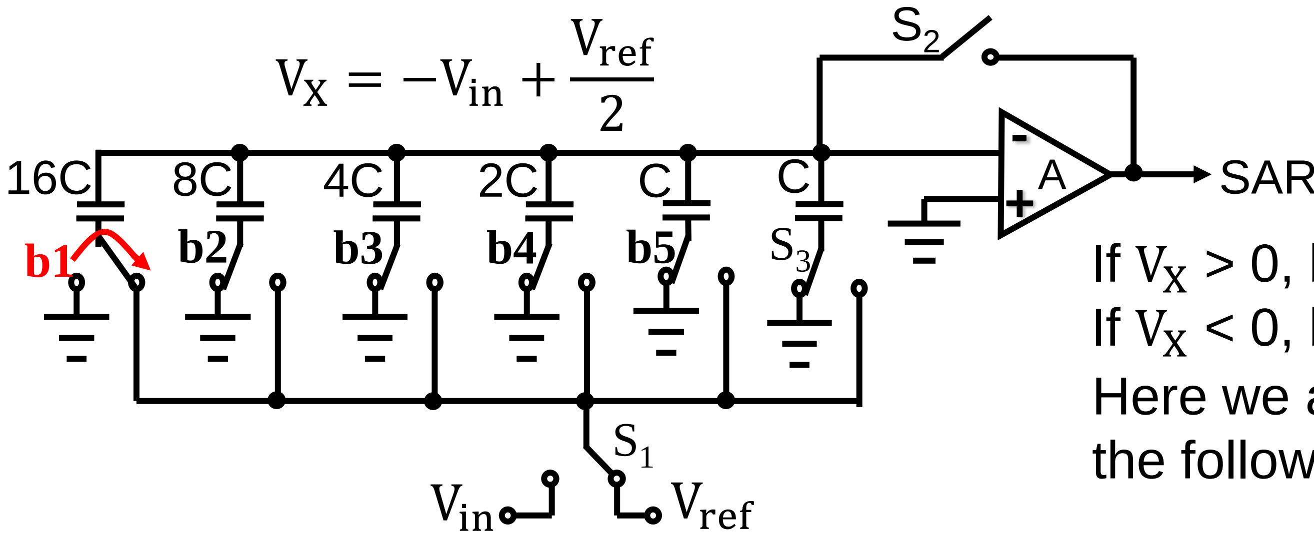
The largest capacitor is switched to V_{ref}



$$Q = 16C \cdot (-V_{in}) + 16C \cdot (-V_{in})$$
$$= -32C \cdot V_{in}$$

$$Q = 16C \cdot (V_X - V_{ref}) + 16C \cdot V_X$$
$$= 16C \cdot (2V_X - V_{ref}) = -32C \cdot V_{in}$$

$$\Rightarrow V_X = -V_{in} + \frac{V_{ref}}{2}$$

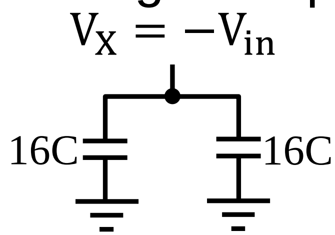


If $V_X > 0$, b1 is determined to be 0
If $V_X < 0$, b1 is determined to be 1
Here we assume b1=1 to explain the following operation

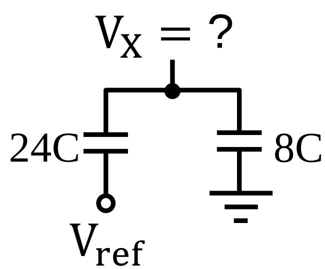
Charge-Redistribution SA ADC (Cont.)

- Assume b1 = 1

The second largest capacitor is switched to V_{ref}

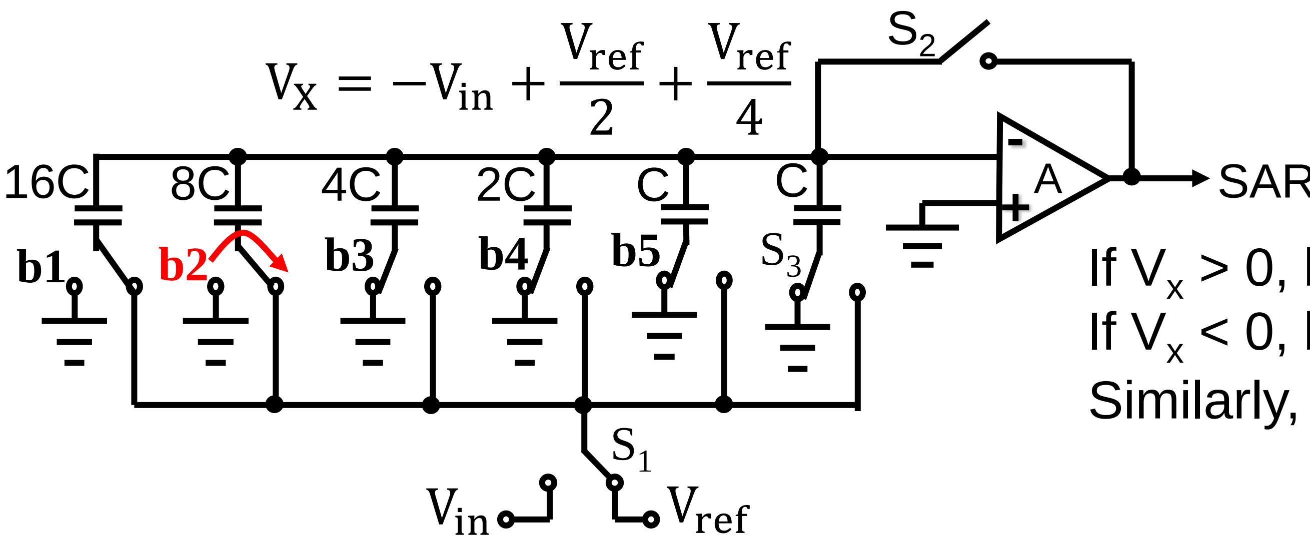


$$Q = 16C \cdot (-V_{in}) + 16C \cdot (-V_{in})$$
$$= -32C \cdot V_{in}$$



$$Q = 24C \cdot (V_X - V_{ref}) + 8C \cdot V_X$$
$$= 32CV_X - 24CV_{ref} = -32C \cdot V_{in}$$

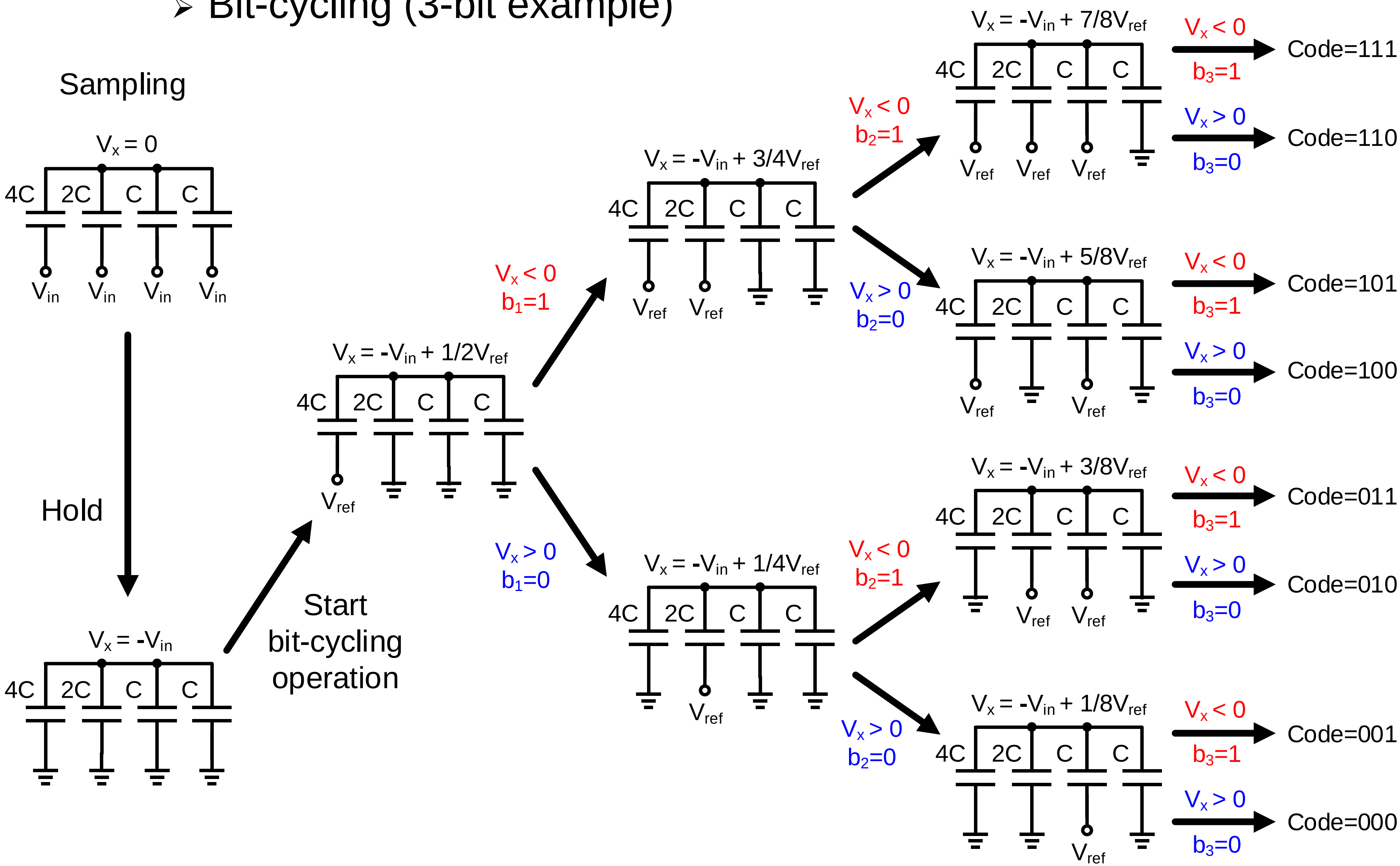
$$\Rightarrow V_X = -V_{in} + \frac{3V_{ref}}{4} = -V_{in} + \frac{V_{ref}}{2} + \frac{V_{ref}}{4}$$



If $V_x > 0$, b2 is determined to be 0
If $V_x < 0$, b2 is determined to be 1
Similarly, b3~b5 can be obtained

Charge-Redistribution SA ADC (Cont.)

- Bit-cycling (3-bit example)



Appendix(II)

- Approximate analysis of feedback amplifier
- General analysis of feedback amplifier
- Multi-stage and multi-loop feedback amplifiers
- Stability using Nyquist diagram
- Frequency response of feedback amplifiers-two-pole system
- Multi-pole Feedback Amplifiers

Approximate Analysis of Feedback Amplifiers

- Assumptions

  1. Basic amplifier
     - Unilateral (input $\rightarrow$ output)
     - Includes loading due to

       a. feedback network

       b. source resistance

       c. load resistance
     - A_o , gain without feedback

  2. Feedback network
     - Unilateral (output $\rightarrow$ input)

Approximate Analysis of Feedback Amplifiers(Cont.)

- Approximation steps

  1. Identify the topology

     one of { series-series, series-shunt, shunt-shunt, shunt-series }

  ➢ Input connection

    a. voltage series

       If $V_i = V_S + V_f$ and $V_f = X_f$

    b. current shunt

       If $I_i = I_s + I_f$ and $I_f = X_f$

  ➢ Output connection

    a. voltage sampling

       set $V_o = 0$ (i.e. set $R_L = 0$) $\Rightarrow$

       if $X_f = 0 \Rightarrow$ voltage sampling $\Rightarrow$ shunt output

    b. current sampling

       set $I_o = 0$ (i.e. set $R_L = \infty$) $\Rightarrow$

       if $X_f = 0 \Rightarrow$ current sampling $\Rightarrow$ series output

Approximate Analysis of A Feedback Amplifiers(Cont.)

2. The basic amplifier configuration without feedback but taking the loading of the feedback network into account
   - Input circuit
     a. Set V_O=0 for a shunt output connection
     b. Set I_O =0 for a series output connection
   - Output circuit
     a. Set V_i=0 for shunt input connection
     b. Set I_i=0 for series input connection
3. Replace each active device by its proper model
4. Identify X_f and X_O on the circuit obtained

Approximate Analysis of A Feedback Amplifiers(Cont.)

5. Evaluate $\beta = \frac{X_f}{X_O}$

6. Evaluate A_O by applying KVL and KCL to the equivalent circuit obtained

7. From A_O and β, find T, and A_f

8. From the equivalent circuit find R_{ID} and R_{OD}

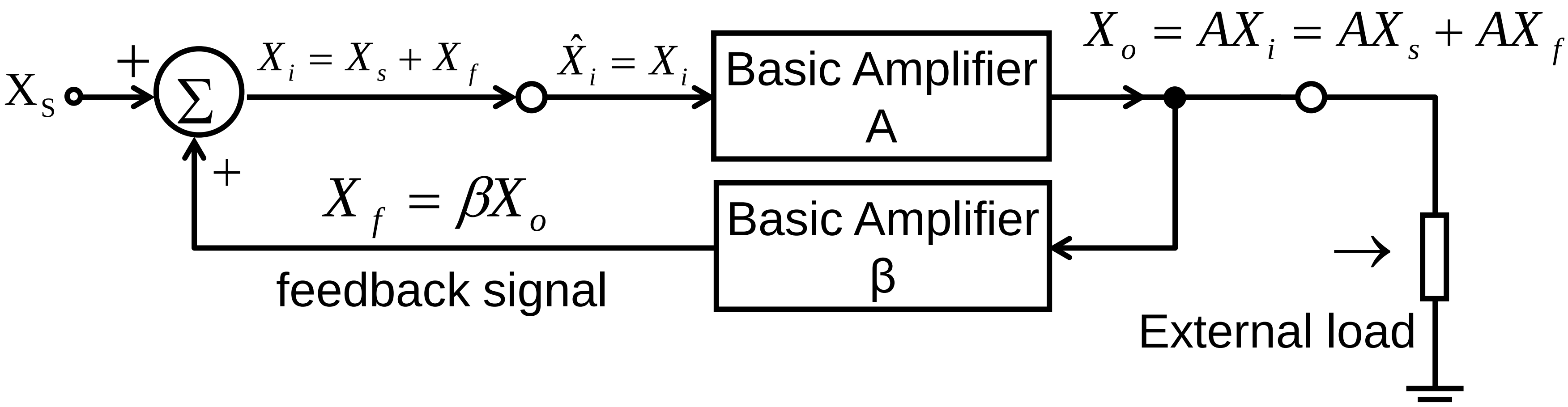
   Apply Blackman's impedance formula to obtain R_{IF} and R_{OF}

General Analysis of Feedback Amplifiers

- Approximate analysis
  - Approximate results
  - Usually accurate enough for hand calculation
  - Results deviate from actual values due to assumptions of unilateral circuits

General Analysis of Feedback Amplifiers (Cont.)

- General analysis
  - Actual results
  - No approximations for amplifier and feedback circuits

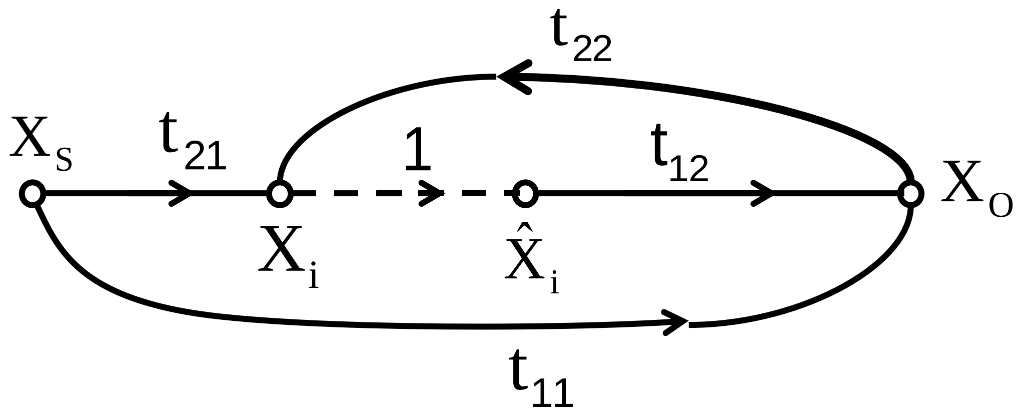


General Analysis of Feedback Amplifiers (Cont.)

$$X_O = t_{11} X_S + t_{12} \hat{X}_i \qquad X_i = t_{21} X_S + t_{22} X_O$$

$$t_{11} = \left. \frac{X_O}{X_S} \right|_{\hat{X}_i = 0} \qquad t_{12} = \left. \frac{X_O}{\hat{X}_i} \right|_{X_S = 0}$$

$$t_{21} = \left. \frac{X_i}{X_S} \right|_{X_O = 0} \qquad t_{22} = \left. \frac{X_i}{X_O} \right|_{X_S = 0}$$



General Analysis of Feedback Amplifiers (Cont.)

- Gain with feedback, A_f
  - $A_f = \frac{X_O}{X_S} = \frac{t_{11} + t_{12}t_{21}}{1 - t_{12}t_{22}}$
  - $A_f\Big|_{t_{12}=0} = \frac{X_O}{X_S}\Big|_{t_{12}=0} \equiv A_D = t_{11}$

  where A_D is the dead-system gain
  - $T = -\frac{X_i}{X_i} = -t_{12}t_{22}$
  - $A_O = \frac{X_O}{X_S}\Big|_{t_{22}=0} = t_{11} + t_{12}t_{21} = A_D + t_{12}t_{21}$
- Impedance is feedback amplifiers
  - The Blackman's impedance formula can be derived using the general analysis

Multi-Stage and Multi-Loop Feedback Amplifiers

- Examples of multistage
  - ◆ Shunt-feedback triple : 3-stage
  - ◆ Shunt-series pair : 2-stage
  - ◆ Series-shunt pair : 2-stage
  - ◆ Series-triple : 3-stage

$\Longrightarrow$ Based on the single-stage analysis (approximate & general), multistage amplifiers can be similarly analyzed.

Multi-Stage and Multi-Loop Feedback Amplifiers (Cont.)

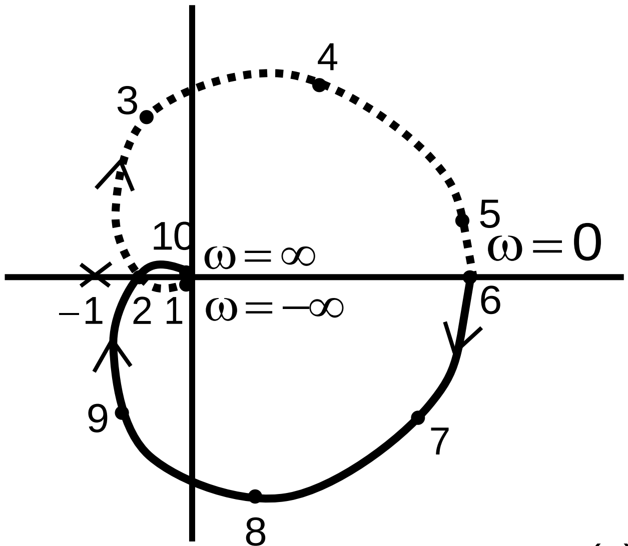
- Examples of multiloop
  - Positive-negative-feedback amplifier two potential drawbacks
    - Sensitive to component variations
    - Potential to cause oscillation due to the use of positive feedback
  - McMillan structure
  - Follow-the-leader feedback
  - Leap-frog feedback

Stability Study Using Nyquist Diagram

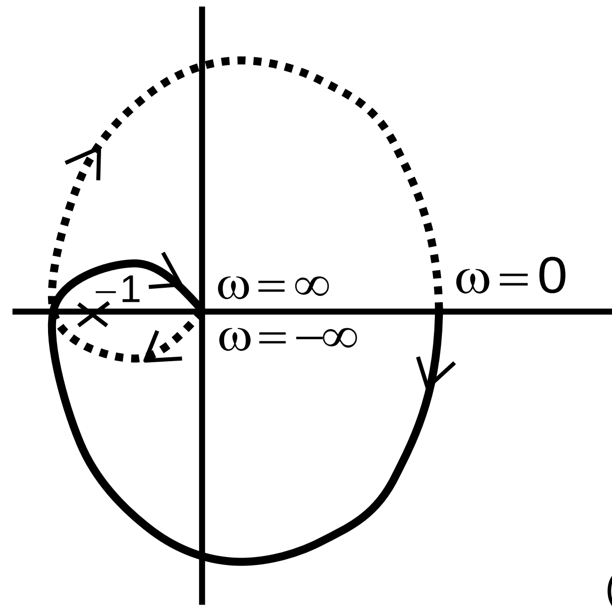
- Nyquist diagram to determine whether a feedback amplifier had any right-half-plane poles
- The Nyquist diagram is a plot of $T(j\omega) = |T(j\omega)| \angle \theta(j\omega)$ in polar coordinates, i.e. at each angular frequency $-\infty < \omega < +\infty$, $T(j\omega)$ and $\theta(j\omega)$ are evaluated.

Stability (Cont.)

- Plots of $T(j\omega) = |T(j\omega)| \angle\theta(j\omega)$



(a)



(b)

Stability (Cont.)

- Nyquist Criterion

  The Nyquist criterion states that the number of clockwise encirclements of the point –1 +j0 equals the difference between the number of zeros and the number of poles of F(s)=1+T(s) in the right half planes. For stability, we must ascertain that F(s) has no right-half-plane zero, that is, stability in feedback amplifiers A_F(s) has no poles of T, and if the amplifier without feedback is stable, F(s) has no poles in the right half plane. Thus, for these conditions, the number of encirclements of –1+j0 must be zero for the feedback amplifier to be stable.

Stability (Cont.)

- Phase margin
  - Gain-crossover angular frequency ω_G

    $|T(j\omega)| > 1$ for $\omega < \omega_G$

    $|T(j\omega)| < 1$ for $\omega > \omega_G$
  - Phase margin Φ_M

    $\Phi_M = \angle T(j\omega_G) + 180°$
  - The closed-loop system is stable when the phase margin is positive (i.e. $\Phi_M > 0$) Hence, $\angle T(j\omega_G)$ must be less negative than $-180°$

Example

Ex : The return ratio of a two-pole amplifier is

$$T(s) = \frac{100}{(1+s/10^6)(1+s/10^7)}$$

(a) Determine the phase margin.

(b) Is the amplifier stable ?

Sol : (a) Asymptotic Bode plot is displayed in the next page, from which $\omega_G = 10^{7.5} = 3.16 \times 10^7 \text{ rad/s}$. On the phase curve, we see that $\angle T = 157.5°$ and

$$\Phi_M = -157.5° + 180° = 22.5°$$

as indicated on the figure at the next page.

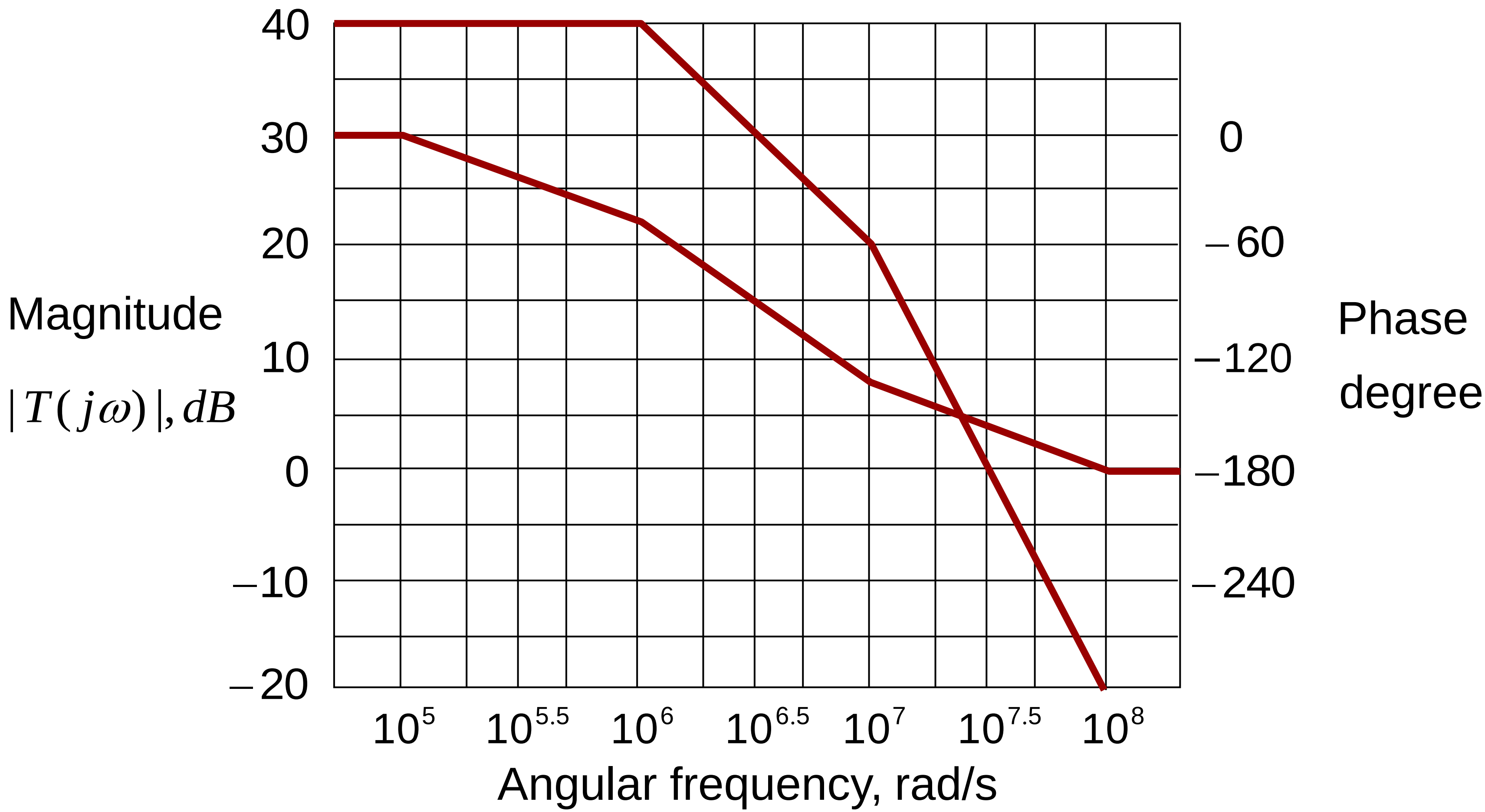
(b) As $\Phi_M > 0°$, the amplifier is stable.

Calculation using the actual Bode diagram, and verified by SPICE tool, gives $\omega_G = 3.09 \times 10^7 \text{ rad/s}$ and $\Phi_M = 20.2°$.

These are in good agreement with the values obtained from the asymptotic Bode diagram.

Example (Cont.)

- Asymptoic Bode diagram



Example (Cont.)

- Note that in the example we cannot identify the phase-crossover frequency and, consequently, the gain margin. This results from the fact that for a two-pole system, the angle is never –180^o but approaches it asymptotically as $\omega \rightarrow \infty$ .Thus we conclude that a two-pole feedback amplifier is always stable. This is also verified by the root locus in the pervious page.

Stability (Cont.)

- Gain margin
  - phase-crossover angular frequency ω_ϕ

$$\angle T < -180^\circ \quad for\ \omega > \omega_\phi$$

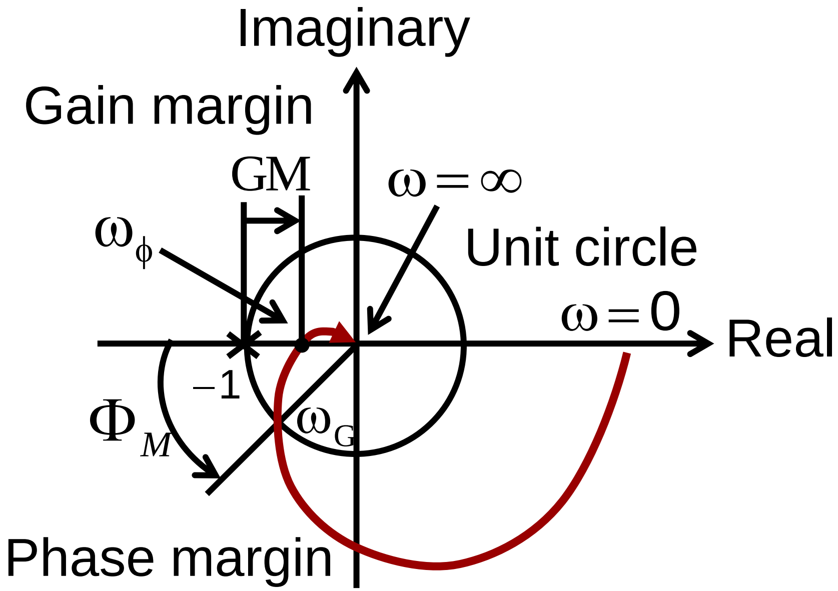
$$\angle T > -180^\circ \quad for\ \omega < \omega_\phi$$

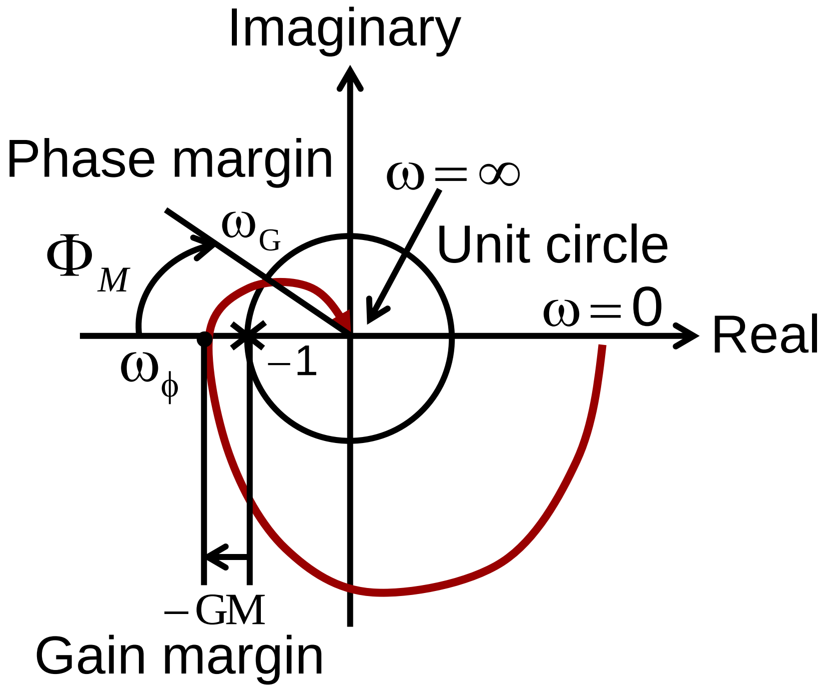
  - gain margin GM

$$\mathrm{GM} = -20\,log\,\mathrm{T}\left(j\omega_\phi\right) = -\mathrm{T}\left(j\omega_\phi\right)\mathrm{dB}$$

Nyquist diagram V.S. Bode diagram

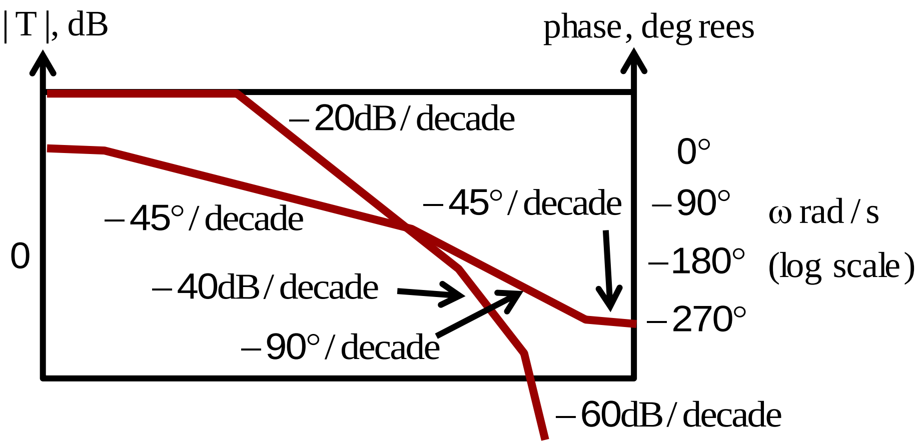
- Nyquist diagram
  - A plot of $T(j\omega) = |T(j\omega)| \angle \theta(j\omega)$

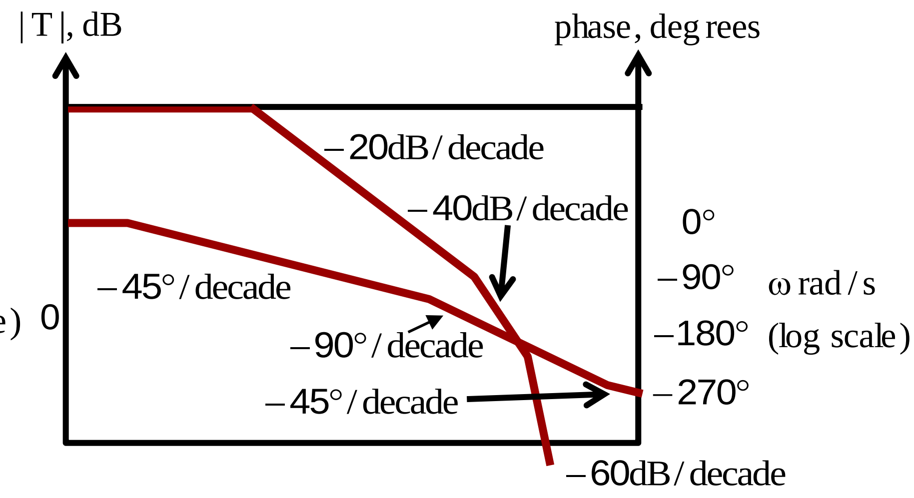




Nyquist diagram V.S. Bode diagram (Cont.)

- Bode diagram





- For a stable system $\Phi_M > 0 \;\&\; GM > 0$
- For a unstable system $\Phi_M < 0 \;\&\; GM < 0$

Frequency Response of Feedback Amplifiers-two-pole system

- $$A_O(s) = \frac{A_0}{1 + s(\frac{1}{\omega_1} + \frac{1}{\omega_2}) + \frac{s^2}{\omega_1\omega_2}} = \frac{A_0}{1 + a_1 s + a_2 s^2}$$

  $$T(s) = \frac{\beta A_0}{1 + a_1 s + a_2 s^2}$$

- $$A_f(s) = \frac{\frac{A_0}{1+\beta A_0}}{1 + \frac{a_1 s}{1+\beta A_0} + \frac{a_2 s^2}{1+\beta A_0}} = \frac{A_{Of}}{1 + \frac{s}{1+\beta A_0}(\frac{1}{\omega_1} + \frac{1}{\omega_2}) + \frac{s^2}{(1+\beta A_0)\omega_1\omega_2}}$$

  $$= \frac{A_{Of}}{1 + (\frac{s}{\omega_0})(\frac{1}{Q}) + (\frac{s}{\omega_0})^2}$$

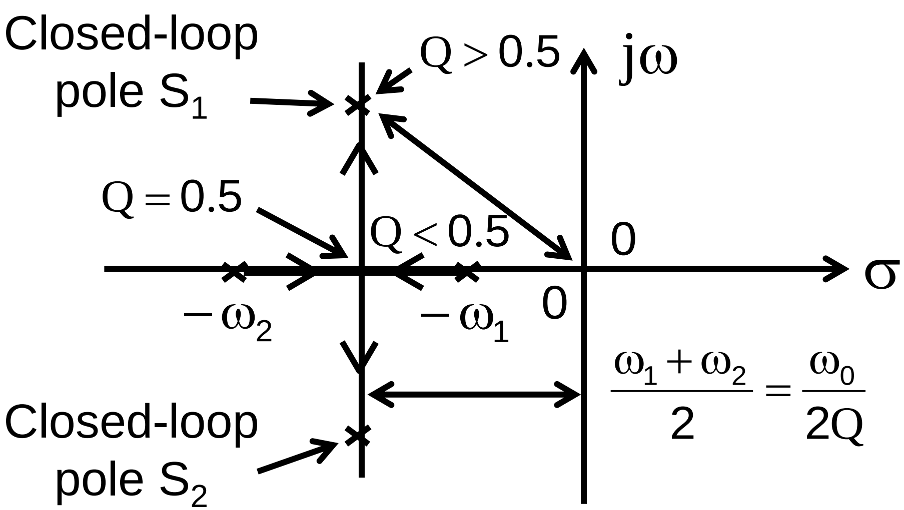
  - ◆ $$\omega_0 = \sqrt{\omega_1\omega_2(1+\beta A_0)} \ \& \ Q = \frac{\omega_0}{\omega_1 + \omega_2}$$

Frequency Response of Feedback Amplifiers-two-pole system (Cont.)

◆ Pole s_1 & s_2 of $A_f(s)$

$$\frac{s}{\omega_0} = -\frac{1}{2Q} \pm \frac{1}{2Q}\sqrt{1-4Q^2}$$

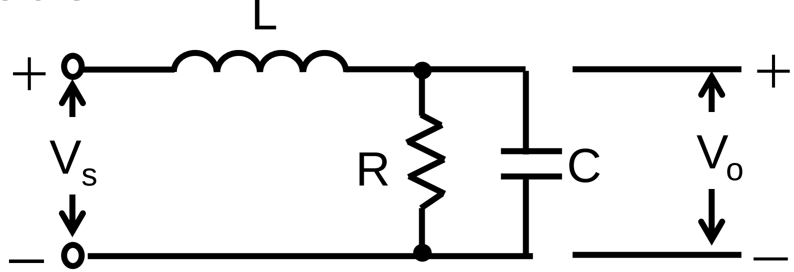
$$s = -\frac{\omega_1+\omega_2}{2} \pm \frac{\omega_1+\omega_2}{2}\sqrt{1-4Q^2}$$



$$\beta A_0 = 0 \Rightarrow \omega_0 = \sqrt{\omega_1\omega_2} \;\&\; Q_{min} = \frac{\sqrt{\omega_1\omega_2}}{\omega_1+\omega_2}$$

Frequency Response of Feedback Amplifiers-two-pole system (Cont.)

- Poles of A_F are
  1. real, negative, and unequal for $Q < 0.5$
  2. real, negative, and equal to $\frac{\omega_1 + \omega_2}{2}$ for $Q = 0.5$
  3. complex conjugate for $Q > 0.5$
- RLC circuit model



  - $\dfrac{V_O(s)}{V_S(s)} = \dfrac{1}{1 + (L/R)s + LCs^2}$
  - $\omega_0 = \dfrac{1}{\sqrt{LC}}$ & $Q = R\sqrt{\dfrac{C}{L}} = \dfrac{R}{\omega_0 L} = \omega_0 RC \Rightarrow \dfrac{V_O(s)}{V_S(s)} = \dfrac{1}{1 + (\frac{s}{\omega_0})(\frac{1}{Q}) + (\frac{s}{\omega_0})^2}$

Frequency Response of Feedback Amplifiers-two-pole system (Cont.)

where ω_0 = undamped ($R \rightarrow \infty$) resonant angular frequency of oscillation

Q = quality factor at the resonant frequency

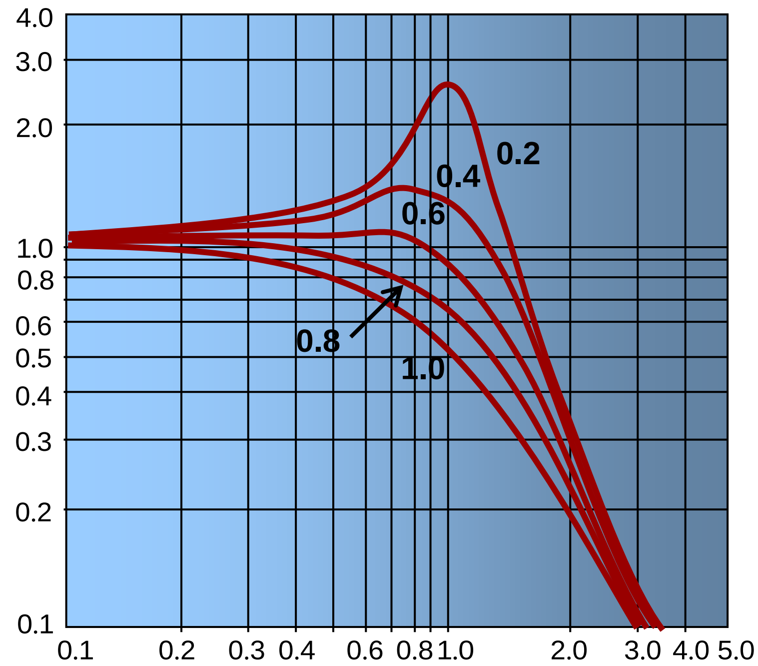
- The response of networks containing resistors, capacitors, and inductors (RLC networks) can be obtained by the use of feedback with circuits that contain only resistors, capacitors, and controlled sources (transistor amplifiers). This is extremely important as inductors cannot be fabricated on an IC.

Frequency Response

- damping factor $k = \dfrac{1}{2Q}$

$$\left|\frac{A_F}{A_{FO}}\right| = \frac{1}{\sqrt{[1-(\omega/\omega_0)^2]^2 + 4k^2(\omega/\omega_0)^2}}$$

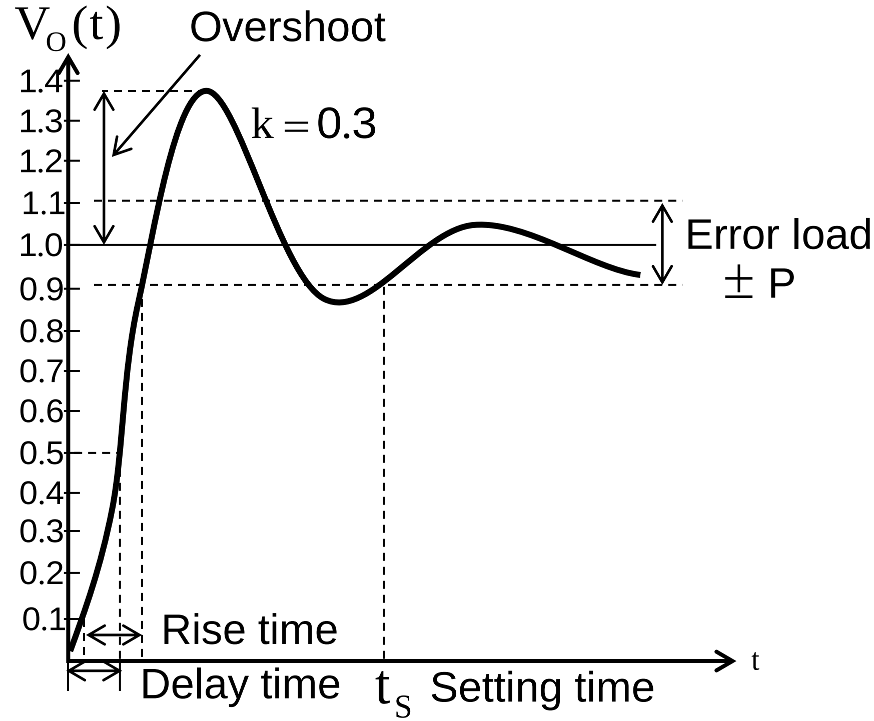
$$\Rightarrow \angle \frac{A_F}{A_{FO}} = -tan^{-1}\frac{2k(\omega/\omega_0)}{1-(\omega/\omega_0)^2}$$



- $k < 0.707$ or $Q > 0.707 = \dfrac{\sqrt{2}}{2}$, peak occurs
- $k > 0.707$ or $Q < 0.707$, peak disappear
- Peak occurs at $\omega = \omega_0\sqrt{1-2k^2}$
- The magnitude of peak $\left|\dfrac{A_F}{A_{FO}}\right|_{peak} = \dfrac{1}{2k\sqrt{1-k^2}}$

Step Response

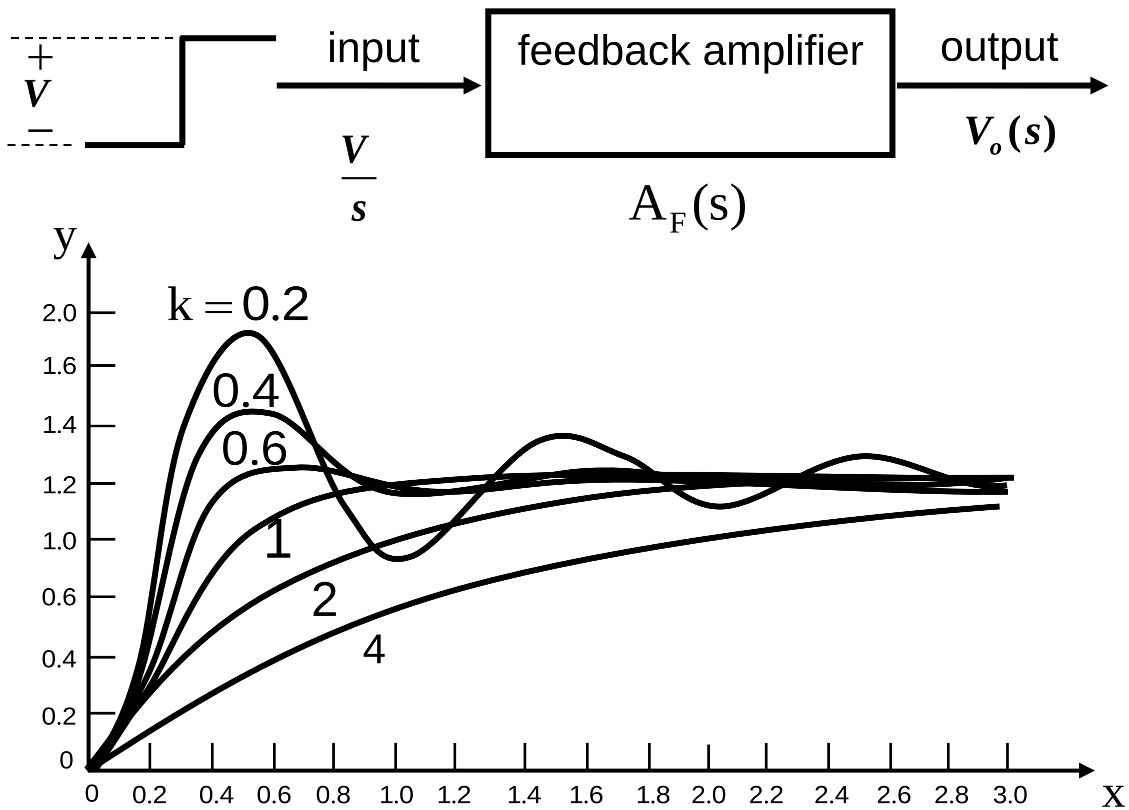
- step response with k = 0.3



The step response of a two-pole feedback amplifier for a damping factor k = 0.3

Rise time= time for waveform to rise from 0.1 to 0.9 of its steady-state value
Delay time = time for waveform to rise from 0 to 0.5 of its steady-state value
Overshoot = peak excursion above the steady-state value
Damped period = time interval for one cycle of oscillation
Settling time = time for response to settle to within±P percent of the steady-state value (P specified for a particular application, say P = 0.1)

Normalized Step Response



- $s_{1,2} = -k\omega_0 \pm \omega_0\sqrt{k^2 - 1}$ $\quad V_O(s) = \frac{V}{s} A_F(s) = \frac{V}{s} \frac{A_{FO}}{s\ \ (1 + s/s_1)(1 + s/s_2)}$

$$\frac{V_O(s)}{VA_{FO}} = \frac{1}{(1 + s/s_1)(1 + s/s_2)}$$

Normalized Step Response (Cont.)

- If k=1, the two poles coincide, corresponding to critically damped case

$$\frac{V_O(t)}{VA_{FO}} = L^{-1}\left[\frac{V_O(s)}{VA_{FO}}\right] = 1-(1+\omega_O t)e^{-\omega_O t}$$

- If k>1, both poles are real and negative, corresponding to an overdamped circuit

$$\frac{V_O(t)}{VA_{FO}} = 1-\frac{1}{2\sqrt{k^2-1}}\left(\frac{1}{k_1}e^{-k_1\omega_0 t}-\frac{1}{k_2}e^{-k_2\omega_0 t}\right)$$

$$\text{where} \quad k_1 = k-\sqrt{k^2-1} \;\&\; k_2 = k+\sqrt{k^2-1}$$

Normalized Step Response (Cont.)

- If k<1, the poles are complex conjugates, corresponding to an underdamped condition

$$\frac{V_O(t)}{VA_{FO}} = 1 - \left(\frac{k\omega_0}{\omega_d} \sin \omega_d t + \cos \omega_d t \right) e^{-k\omega_0 t}$$

$$\text{where} \quad \omega_d = \sqrt{1-k^2}\,\omega_0$$

Phase Margin of the Two-pole Feedback Amplifier

- Recall that

$$\omega_0 = \sqrt{\omega_1\omega_2(1+\beta A_O)}\ \&\ Q = \frac{\omega_0}{\omega_1+\omega_2}$$

$$s = -\frac{\omega_1+\omega_2}{2} \pm \frac{\omega_1+\omega_2}{2}\sqrt{1-4Q^2}$$

  - ◆ Pole-separation factor

$$\omega_0 = \omega_1\sqrt{n(1+\beta A_O)}\ \&\ Q = \frac{\sqrt{n(1+\beta A_O)}}{n+1}$$

$$s = -\frac{\omega_1(1+n)}{2}(1\pm\sqrt{1-4Q^2})$$

$$n = \frac{\omega_2}{\omega_1}$$

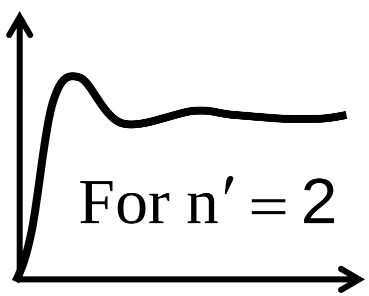
Phase Margin of the Two-pole Feedback Amplifier (Cont.)

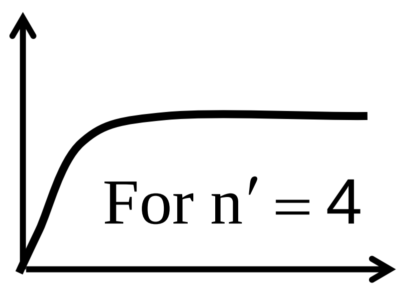
If $n >> 1 \Rightarrow n \approx \frac{1+\beta A_O}{Q^2}$

$$n' = \frac{n}{\beta A_0} = \frac{1}{Q^2}$$

$$\frac{\omega_2}{\omega_1} = n \qquad \frac{\omega_2}{\omega_t} = n'$$

$Q = \frac{1}{2}$,
- $n' = 2$ (fast) — Phase margin=63°
- $n' = 4$ (critically damped) — Phase margin=76°
- $n' = 3$ — Phase margin=71°





Phase Margin of the Two-pole Feedback Amplifier (Cont.)

- Phase margin Φ_M

$$T(s) = \frac{\beta A_O}{(1+s/\omega_1)(1+s/n\omega_2)}$$

  - Let $T(j\omega_G) = 1$

$$\Rightarrow \frac{\omega_G}{\omega_1} = \sqrt{\frac{n^2+1}{2}}\left[\sqrt{\frac{4n^2((\beta A_O)^2-1)}{(n^2+1)^2}+1}-1\right]^{1/2}$$

Where ω_G is the angular gain crossover frequency

  - For $n^2 >> 1, (\beta A_O)^2 >> 1$, and $n \approx \dfrac{1+\beta A_O}{Q^2}$

$$\Rightarrow \frac{\omega_G}{\omega_1} = \frac{\beta A_0}{Q^2\sqrt{2}}(\sqrt{4Q^4+1}-1)^{1/2} = \frac{n}{\sqrt{2}}(\sqrt{4Q^2+1}-1)^{1/2}$$

Phase Margin of the Two-pole Feedback Amplifier (Cont.)

- phase margin $\Phi_M = \angle T(j\omega_G) + 180°$

$$\Rightarrow \Phi_M = -\tan^{-1}\frac{\omega_G}{\omega_1} - \tan\frac{\omega_G}{n\omega_1} + 180°$$

$$\Phi_M = \left(90 - \tan^{-1}\frac{\omega_G}{\omega_1}\right) + \left(90 - \tan^{-1}\frac{\omega_G}{n\omega_1}\right) = \tan^{-1}\frac{\omega_1}{\omega_G} + \tan^{-1}\frac{n\omega_1}{\omega_G}$$

- ◆ Since $\omega_1 << \omega_G \Rightarrow \tan^{-1}\frac{\omega_1}{\omega_G}$ is very small

$$\Phi_M \approx \tan^{-1}\frac{n\omega_1}{\omega_G} = \tan^{-1}\frac{\omega_2}{\omega_G}$$

$$\Phi_M \approx \tan^{-1}\sqrt{2}\left(\sqrt{4Q^2+1} - 1\right)^{-1/2}$$

Phase Margin of the Two-pole Feedback Amplifier (Cont.)

- Closed-loop bandwidth ω_H

  - $A_F\left(j\omega_H\right)\dfrac{A_{FO}}{\sqrt{2}}$

$$\Rightarrow \left\{ \begin{aligned} \omega_H &= \frac{\omega_O}{Q}\sqrt{\frac{2Q^2-1}{2}}\left[1+\sqrt{1+\frac{4Q^4}{\left(2Q^2-1\right)^2}}\right]^{1/2} & Q^2 > 0.5 \\ &= \frac{\omega_O}{Q}\sqrt{\frac{1-2Q^2}{2}}\left[\sqrt{1+\frac{4Q^4}{\left(1-2Q^2\right)^2}}-1\right]^{1/2} & Q^2 < 0.5 \\ &= \omega_0 & Q^2 = 0.5 \end{aligned} \right\}$$

Multi-pole Feedback Amplifiers

- It can be approximated as a two-pole system.
- The accuracy of this approximation is usually sufficient for pencil-and-paper calculations needed to obtain initial design values. As is almost always true, final design values are based on computer analysis.