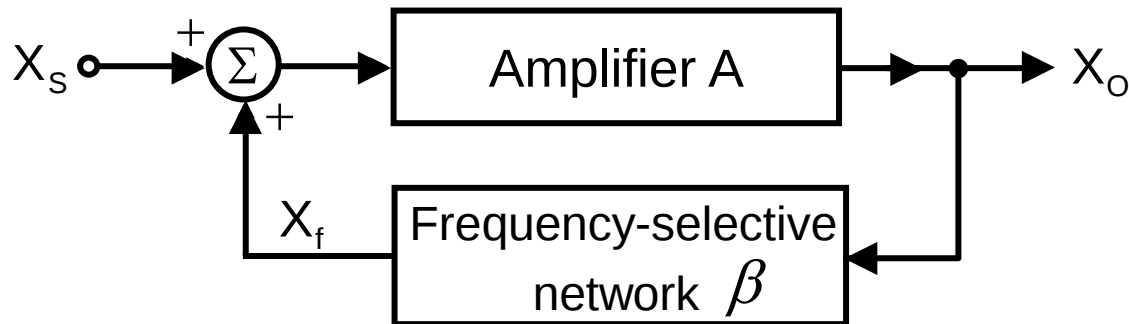


Basic Principles of Sinusoidal Oscillators

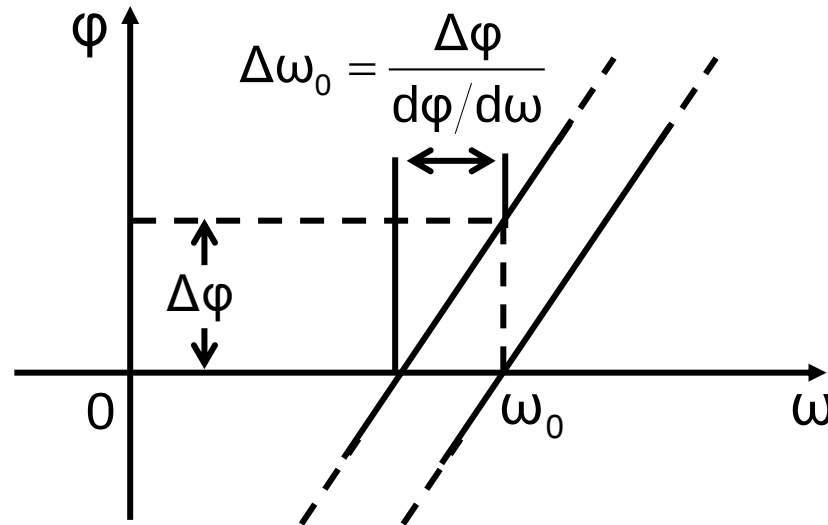
- Linear oscillator
 - ◆ Linear region of circuit: linear oscillation
 - ◆ Nonlinear region of circuit: amplitudes stabilization
- Barkhausen criterion



- ◆ Loop gain $L(s) = \beta(s)A(s)$
- ◆ Characteristic equation $1-L(s) = 0$
- ◆ Oscillation criterion $L(j\omega_0) = A(j\omega_0) \beta(j\omega_0) = 1$
- ◆ At ω_0 , the phase of the loop should be zero and the magnitude of the loop gain should be unity

Basic Principles of Sinusoidal Oscillators (Cont.)

◆ Frequency-phase relation



- Oscillation frequency ω_0 is determined by $\phi(\omega_0)=0^\circ$ (or 360°)
- A steep phase response results in a small $\Delta\omega_0$ for a given change in phase $\Delta\phi$

Nonlinear Amplitude Control

- To sustain oscillation: $\beta A \geq 1$
 - a. Overdesign for βA variations
 - b. Oscillation will grow in amplitude
Poles are in the right half of the s-plane
 - c. Nonlinear network reduces βA to 1 when the desired amplitude is reached.
Poles will be pulled to $j\omega$ -axis

OPAMP-RC Oscillator Circuits

- Wien-bridge oscillator

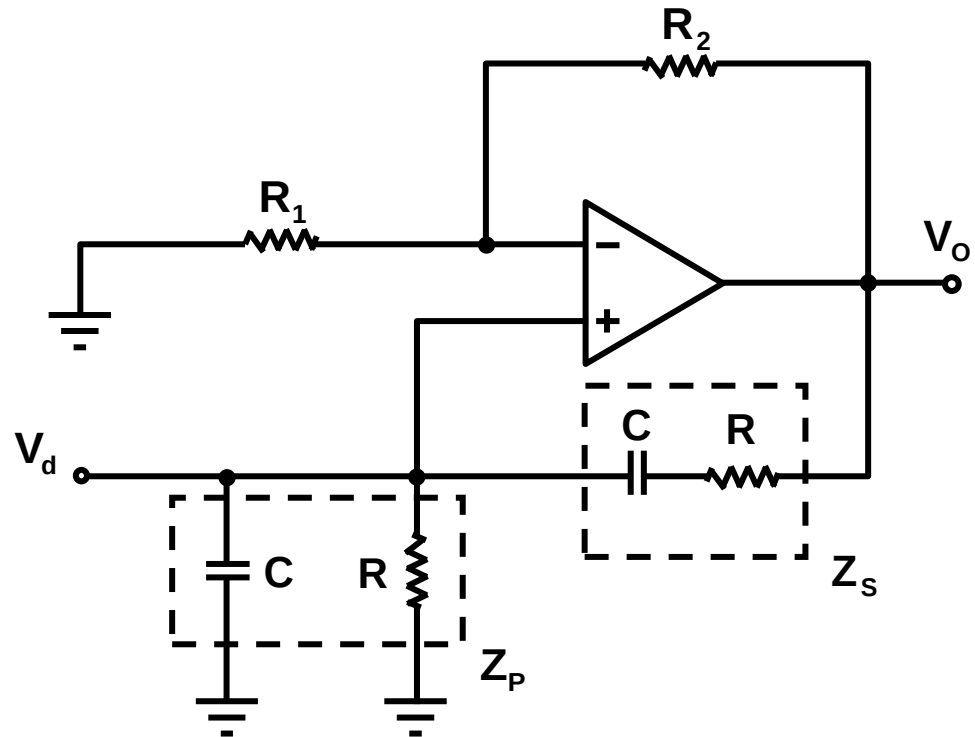
$$L(s) = \left[1 + \frac{R_2}{R_1} \right] \frac{Z_p}{Z_p + Z_s} = \frac{1 + \frac{R_2}{R_1}}{3 + SCR + \frac{1}{SCR}}$$

$$L(j\omega) = \frac{1 + \frac{R_2}{R_1}}{3 + j\left(\omega RC - \frac{1}{\omega RC}\right)}$$

$$\text{For phase} = 0, \omega_0 RC = \frac{1}{\omega_0 RC}$$

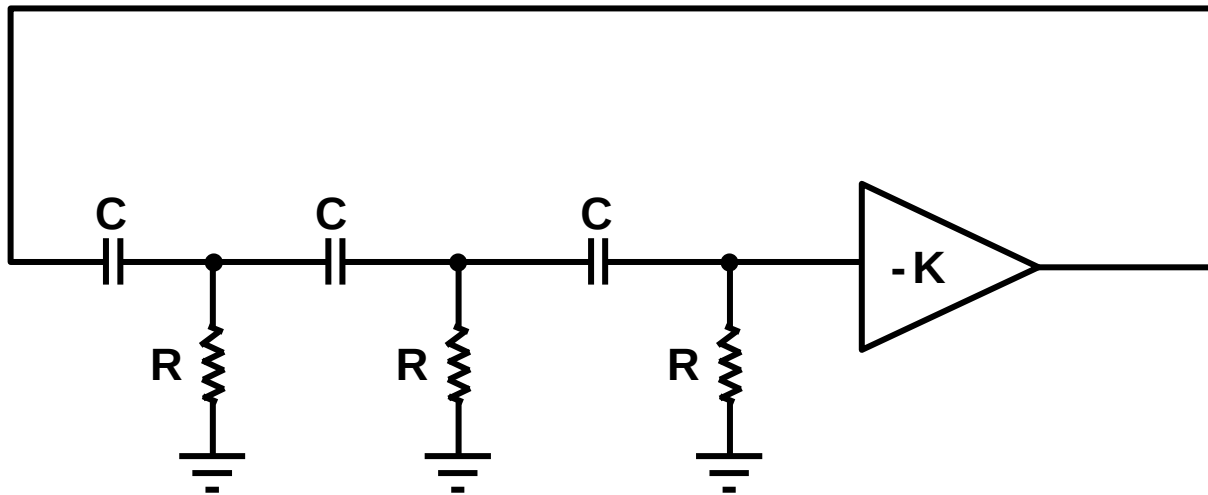
$$\Rightarrow \omega_0 = \frac{1}{RC}$$

$$L(s) = 1 \Rightarrow \frac{R_2}{R_1} = 2$$



Phase-Shift Oscillator

- Without amplitude stabilization

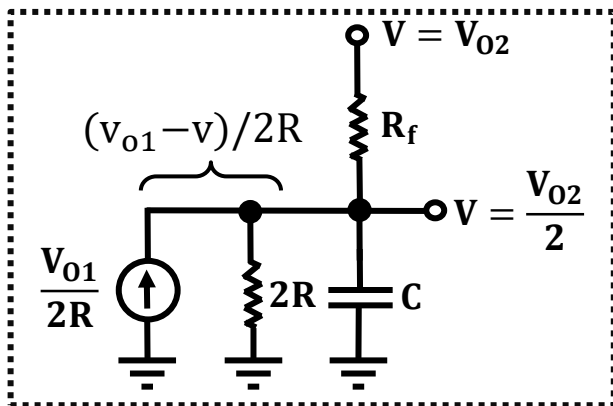


----- phase shift of the RC network is 180 degrees.

⇒ Total phase shift around the loop is 0 or 360 degrees.

Quadrature Oscillator

- OP_1 : inverting integrator ; OP_2 : noninverting integrator
- Equivalent circuit at the input of OP_2

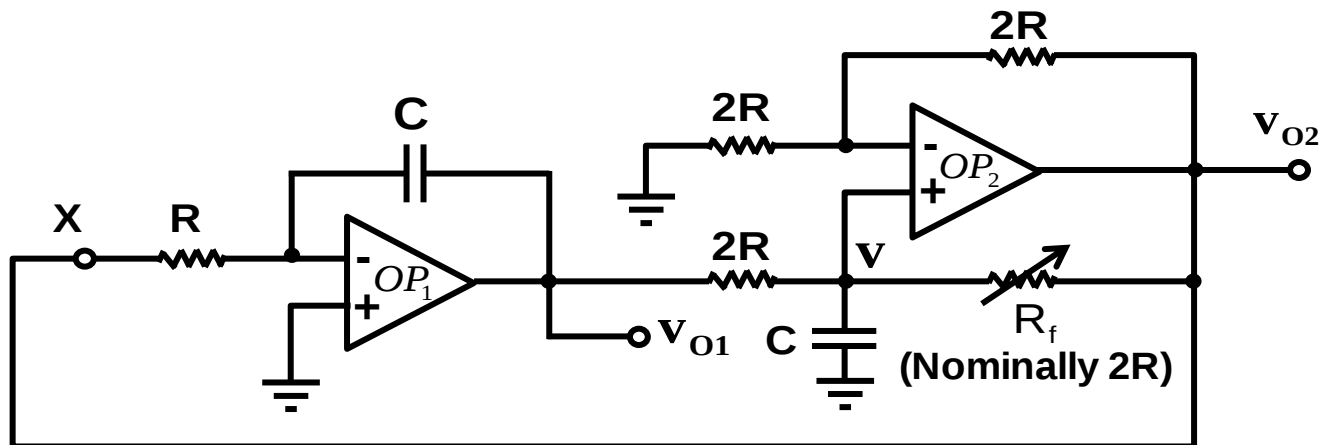


Set $R_f = 2R$

$$\Rightarrow v(t) = \frac{1}{C} \int_0^t \frac{v_{O1}(t)}{2R} dt$$

$$\Rightarrow v_{O2}(t) = \frac{1}{C} \int_0^t \frac{v_{O1}(t)}{R} dt$$

$$\Rightarrow v_{O2}(s) = \frac{1}{sCR} v_{O1}(s)$$



Quadrature Oscillator (Cont.)

----- Break the loop at X, loop gain

$$L(s) = \frac{V_{o2}(s)}{V_x(s)} = -\frac{1}{sCR} \cdot \frac{1}{sCR} = -\frac{1}{s^2 C^2 R^2}$$

$$L(j\omega) = \frac{1}{\omega^2 C^2 R^2}$$

$$\Rightarrow \text{oscillation frequency} \quad \omega_0 = \frac{1}{RC}$$

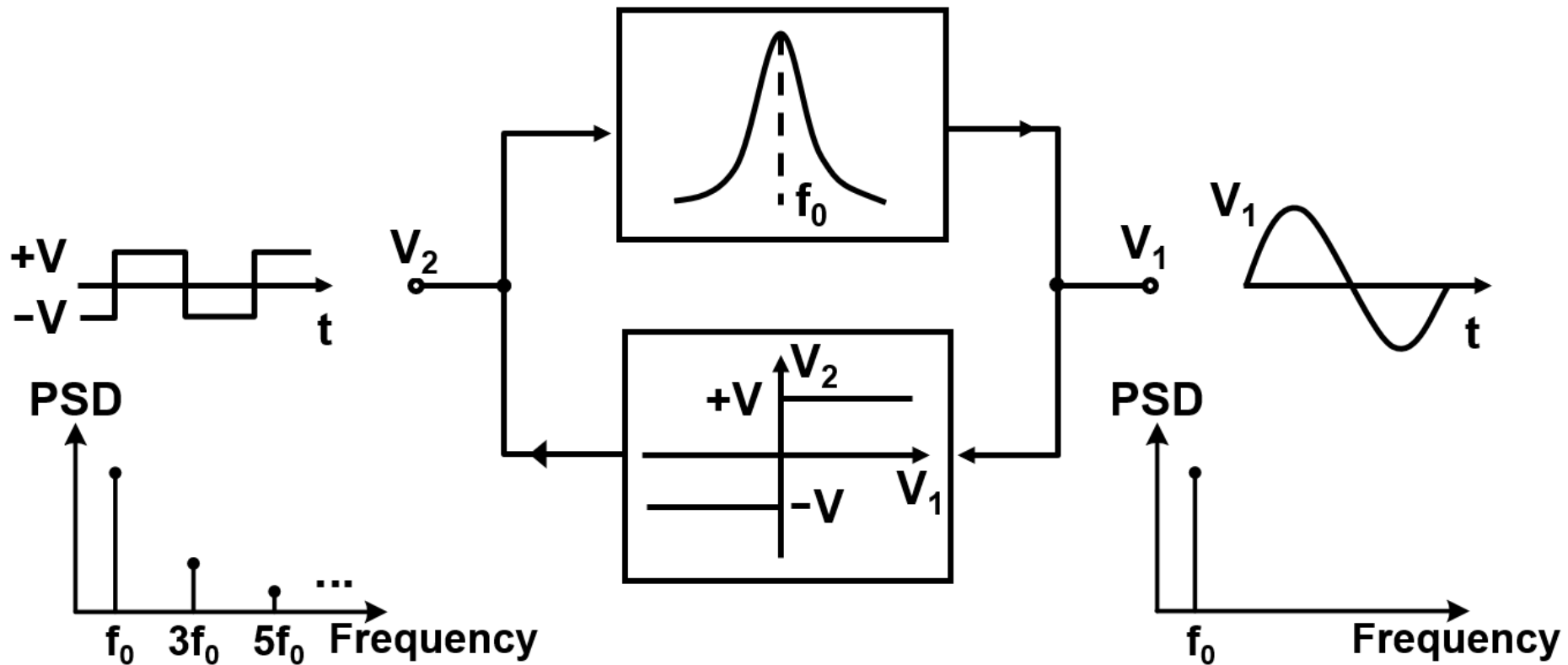
----- V_{o2} is the integral of V_{o1}

$\Rightarrow$ 90° phase difference between V_{o1} and V_{o2}

$\Rightarrow$ “quadrature” oscillator

Reading Assignment: Active-Filter Tuned Oscillator

- Block diagram

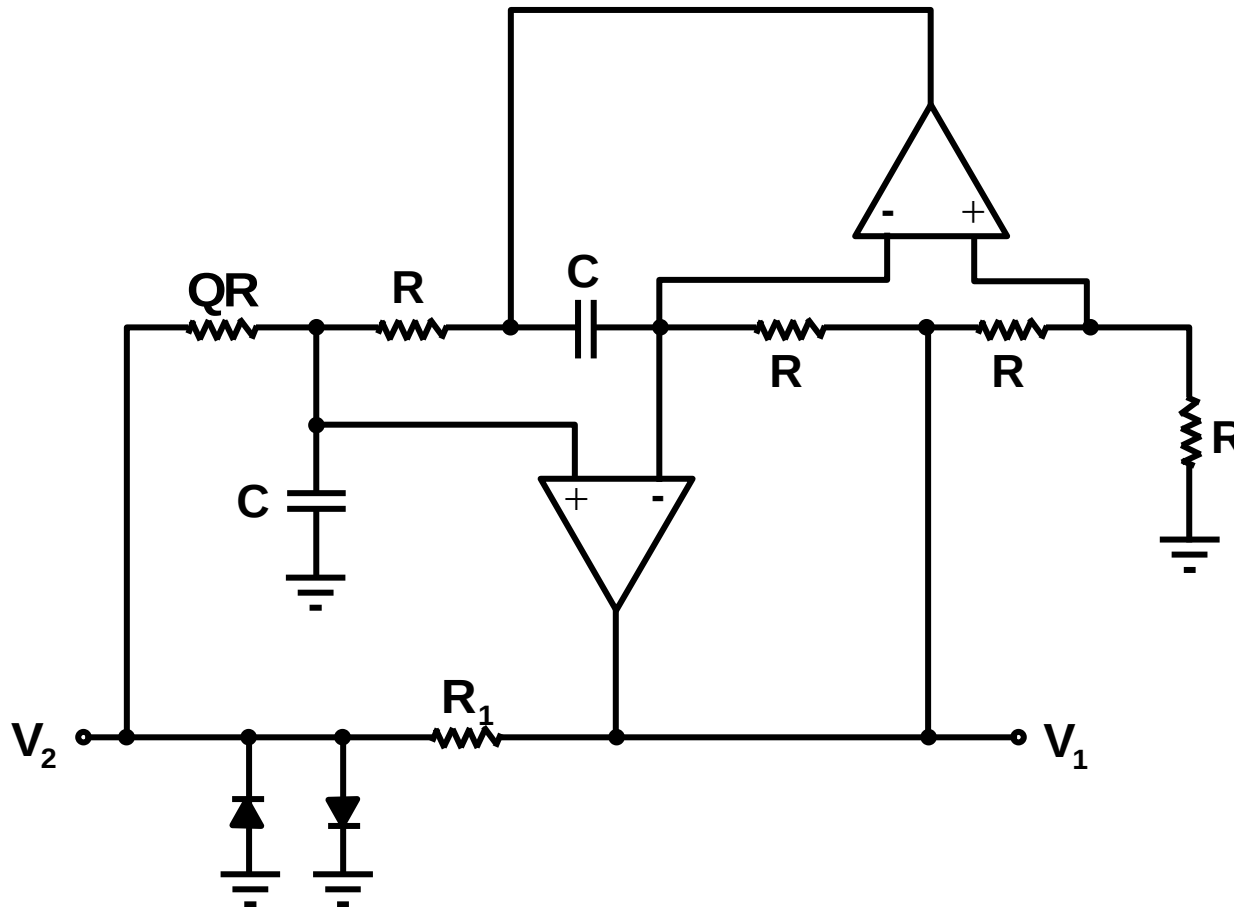


- ◆ High-distortion v_2

- ◆ High-Q bandpass $\Rightarrow$ low-distortion v_1

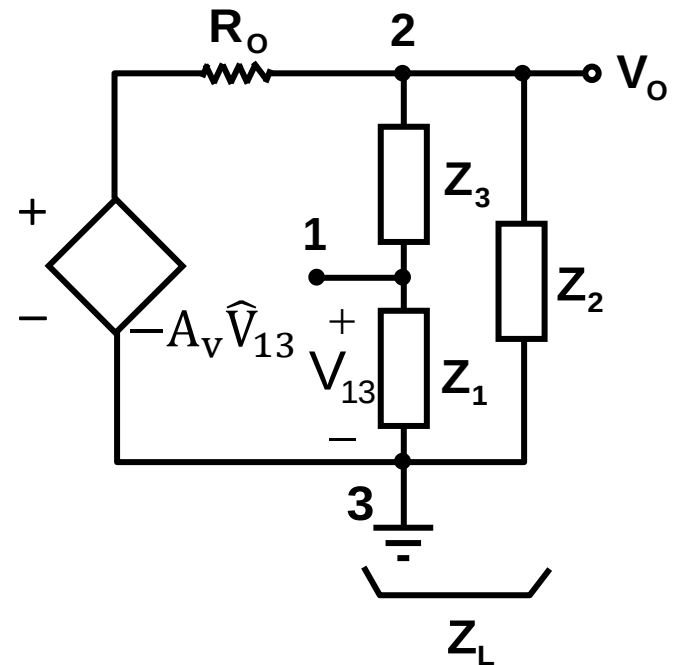
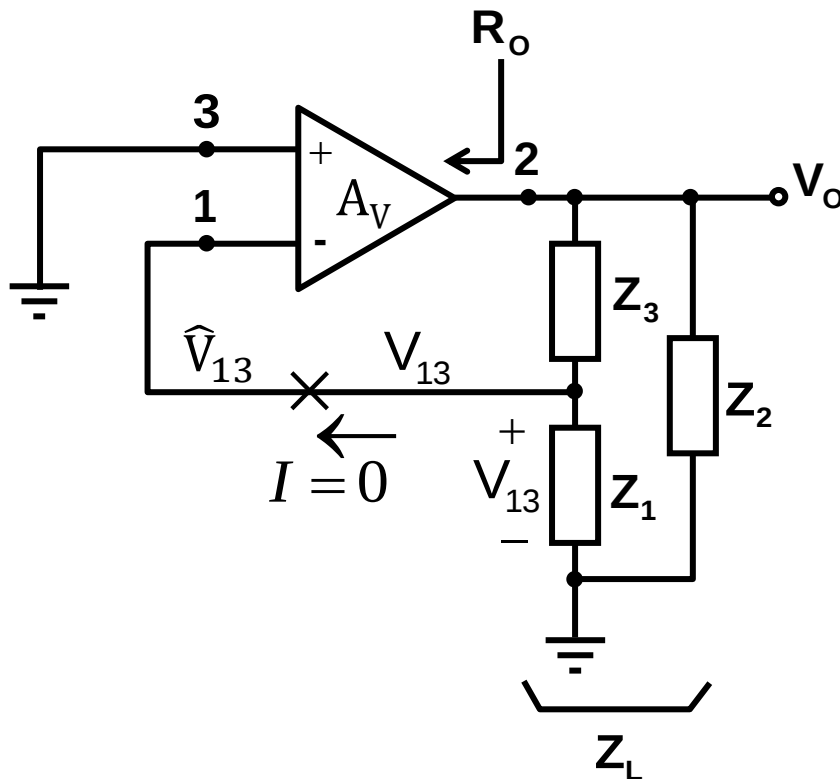
Reading Assignment: Active-Filter Tuned Oscillator (Cont.)

- Practical implementation



A General Form of LC-Tuned Oscillator Configuration

- Many oscillator circuits fall into a general form shown below



- Z_1, Z_2, Z_3 : capacitive or inductive

A General Form of LC-Tuned Oscillator Configuration (Cont.)

$$V_O = \frac{-A_V \hat{V}_{13} Z_L}{Z_L + R_O} \Rightarrow \hat{V}_{13} = -V_O \frac{Z_L + R_O}{A_V Z_L}$$

$$V_{13} = \frac{Z_1}{Z_1 + Z_3} V_O, T = \frac{V_{13}}{\hat{V}_{13}} = \frac{-A_V Z_1 Z_2}{R_O(Z_1 + Z_2 + Z_3) + Z_2(Z_1 + Z_3)}$$

If $Z_1 = jX_1$, $Z_2 = jX_2$, $Z_3 = jX_3$

$X = \omega L$ for inductance $X = -\frac{1}{\omega C}$ for capacitance

$$T = \frac{A_V X_1 X_2}{jR_O(X_1 + X_2 + X_3) - X_2(X_1 + X_3)}$$

for oscillation, $T = 1 \angle 0^\circ$

$$\Rightarrow X_1 + X_2 + X_3 = 0$$

$$\Rightarrow T = \frac{A_V X_1 X_2}{-X_2(X_1 + X_3)} = \frac{-A_V X_1}{X_1 + X_3}$$

$$\Rightarrow T = \frac{A_V X_1}{X_2}$$

A General Form of LC-Tuned Oscillator Configuration (Cont.)

- With oscillation

$|T| = 1$ and $\angle T = 0, 360, 720, \dots$ degree.

i.e. $T = 1$ $(X = \omega L \text{ or } X = -\frac{1}{\omega C})$

$\Rightarrow X_1$ & X_2 must have the same sign if A_v is positive

$\Rightarrow X_1$ & X_2 are L, $X_3 = -(X_1 + X_2)$ is C

or X_1 & X_2 are C, $X_3 = -(X_1 + X_2)$ is L

- Transistor oscillators

1. Collpitts oscillator

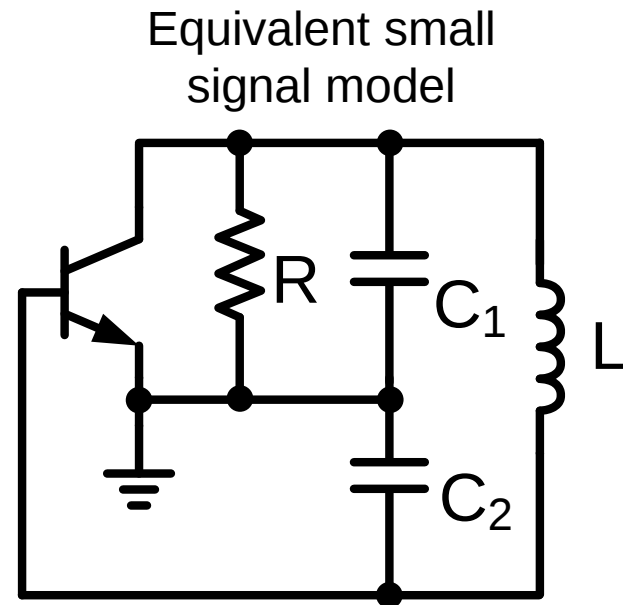
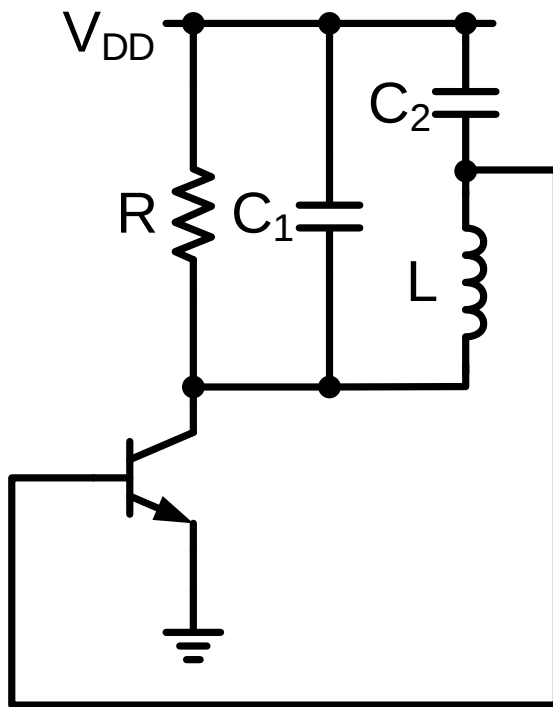
-- X_1 & X_2 are Cs, X_3 is L

2. Hartley oscillator

-- X_1 & X_2 are Ls, X_3 is C

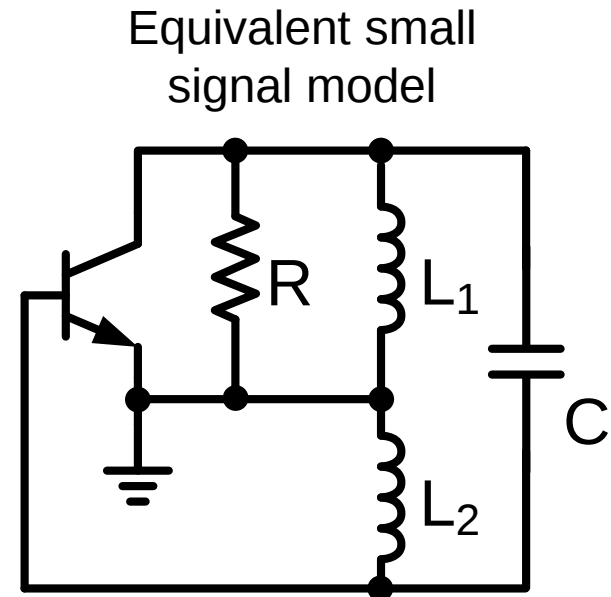
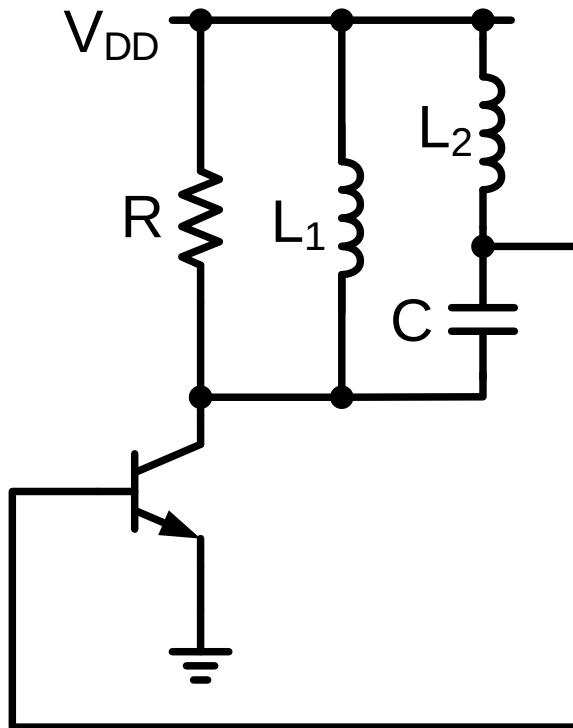
LC-Tuned Oscillators

- Two commonly used configurations
 - ◆ Colpitts (feedback is achieved by using a capacitive divider)



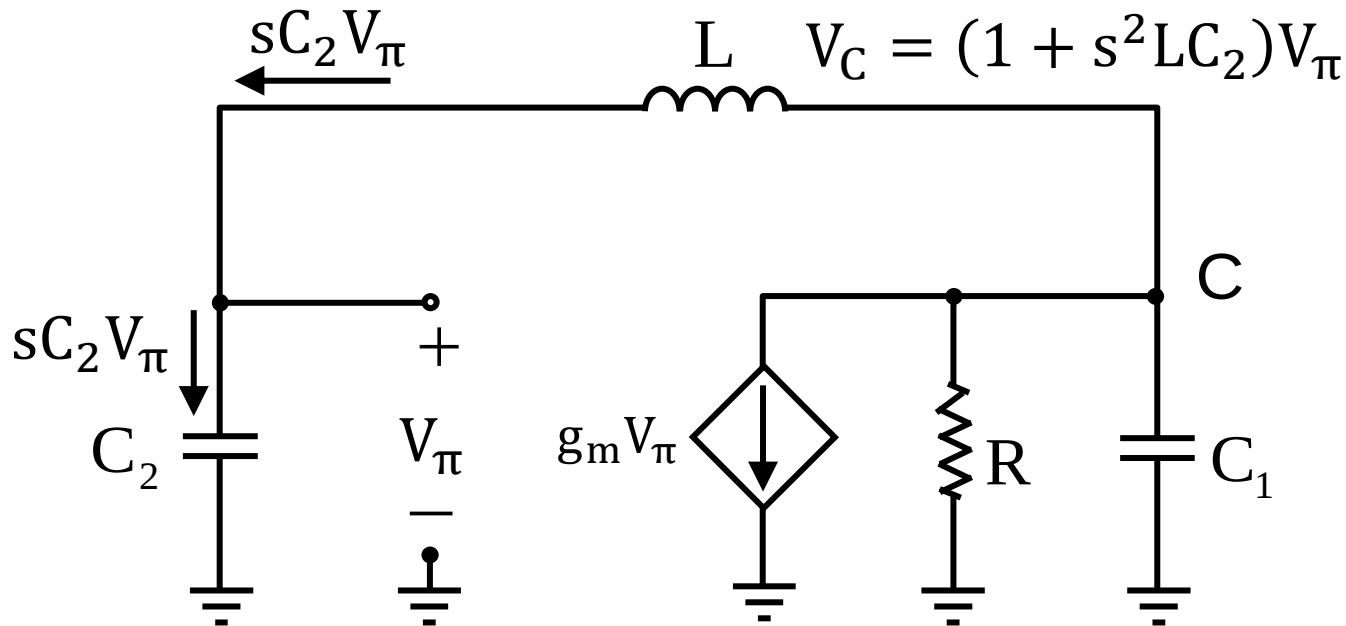
LC-Tuned Oscillators (Cont.)

- ◆ Hartley (feedback is achieved by using an inductive divider)



LC-Tuned Oscillators (Cont.)

- Colpitts oscillator
 - ◆ Equivalent circuit



- ◆ C_π (C_{gs} for a FET) can be considered to be a part of C_2
- ◆ R = loss of inductor + load resistance of oscillator + output resistance of transistor

LC-Tuned Oscillators (Cont.)

$$V_C = (1 + sC_2 \cdot sL)V_\pi$$

$$sC_2 V_\pi + g_m V_\pi + \left(\frac{1}{R} + sC_1 \right) (1 + s^2 LC_2) V_\pi = 0$$

$$s^3 LC_1 C_2 + s^2 \left(\frac{LC_2}{R} \right) + s(C_1 + C_2) + \left(g_m + \frac{1}{R} \right) = 0$$

 $s = j\omega$

$$\left(g_m + \frac{1}{R} - \frac{\omega^2 LC_2}{R} \right) + j[\omega(C_1 + C_2) - \omega^3 LC_1 C_2] = 0$$

- ◆ Both the real and imaginary parts must be zero
- ◆ Oscillation frequency

$$\omega_0 = \frac{1}{\sqrt{L \left(\frac{C_1 C_2}{C_1 + C_2} \right)}}$$

LC-Tuned Oscillators (Cont.)

◆ Gain

$$g_m R = \frac{C_2}{C_1} \quad (\text{Actually, } g_m R \geq \frac{C_2}{C_1})$$

◆ Oscillation amplitude

1. LC tuned oscillators are known as self – limiting oscillators. (As oscillations grown in amplitude, transistor gain is reduced below its small–signal value)
2. Output voltage signal will be a sinusoid of high purity because of the filtering action of the LC tuned circuit

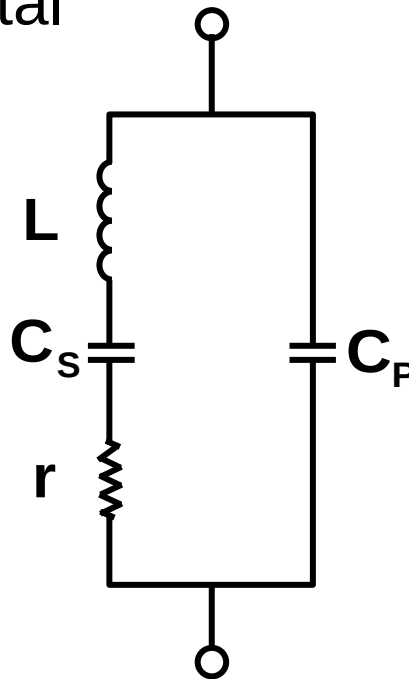
- Hartley oscillator can be similarly analyzed

Crystal Oscillators

- Symbol of crystal



- Circuit model of crystal



Crystal Oscillators (Cont.)

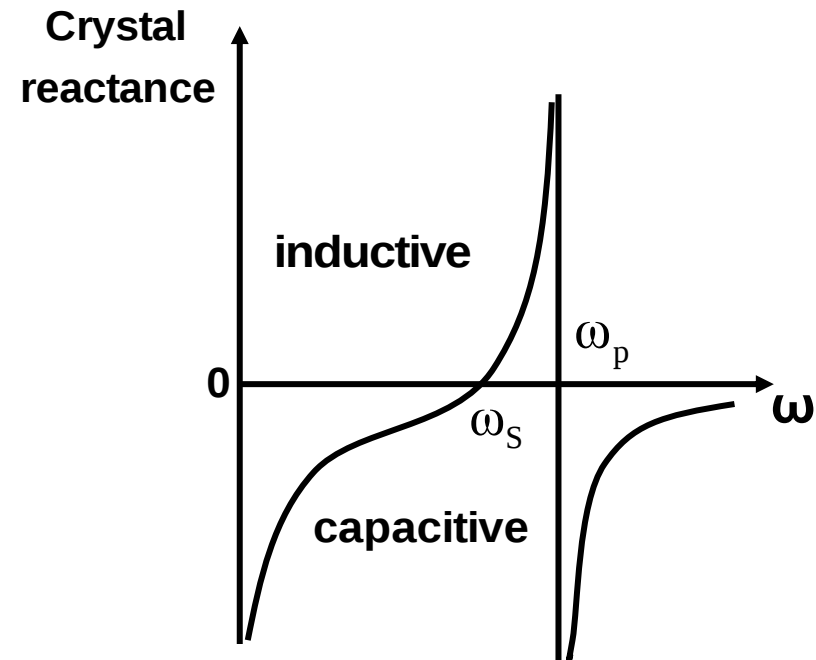
- Reactance of a crystal assuming $r = 0$. Crystal is high-Q

$$Z(s) = \frac{1}{sC_p + \frac{1}{sL + \frac{1}{sC_s}}} = \frac{1}{sC_p} \frac{s^2 + \frac{1}{LC_s}}{s^2 + \frac{(C_p + C_s)}{LC_s C_p}}$$

$$\text{Let } \begin{cases} \omega_s^2 = \frac{1}{LC_s} \\ \omega_p^2 = \frac{1}{L} \left(\frac{1}{C_s} + \frac{1}{C_p} \right) \end{cases}$$

$$\Rightarrow Z(j\omega) = -j \frac{1}{\omega C_p} \left(\frac{\omega^2 - \omega_s^2}{\omega^2 - \omega_p^2} \right)$$

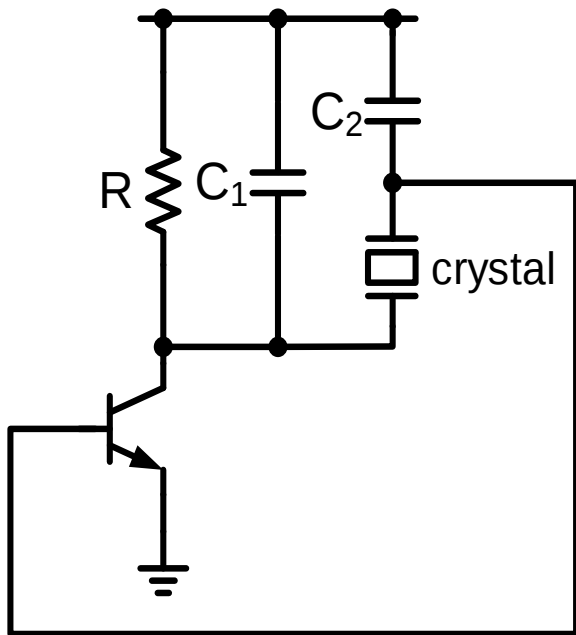
If $C_p \gg C_s$, then $\omega_p \approx \omega_s$



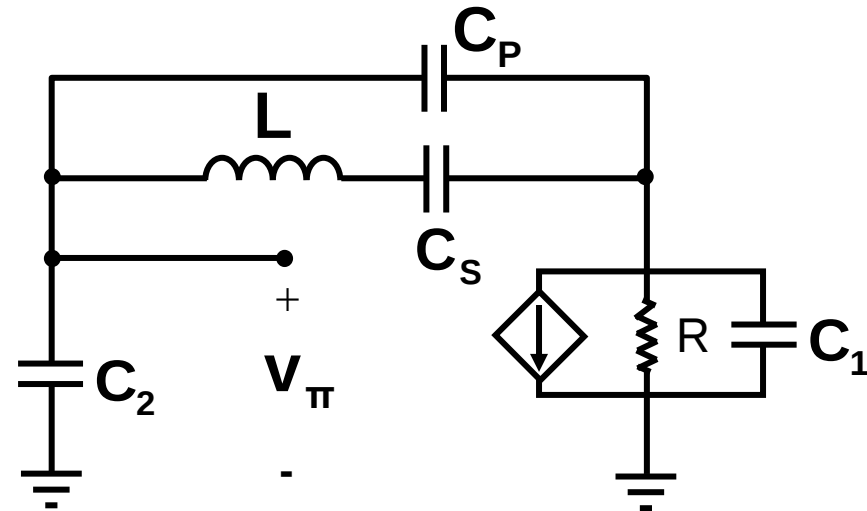
Crystal Oscillators (Cont.)

- The crystal reactance is inductive over the narrow frequency band between ω_s and ω_p
- Colpitts crystal oscillator

◆ Configuration



◆ Equivalent circuit



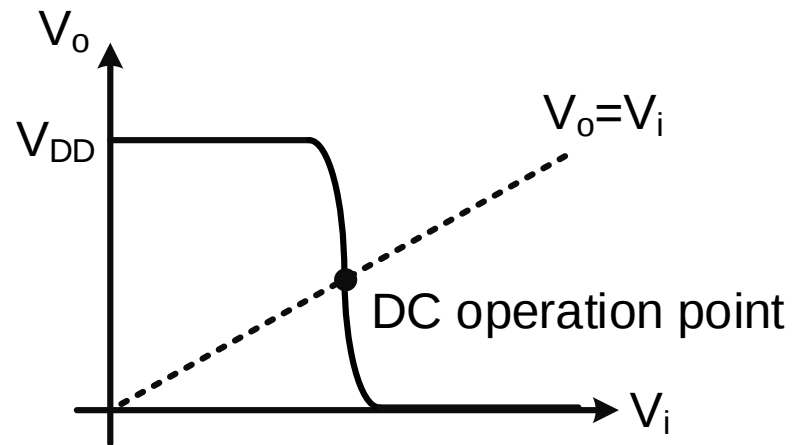
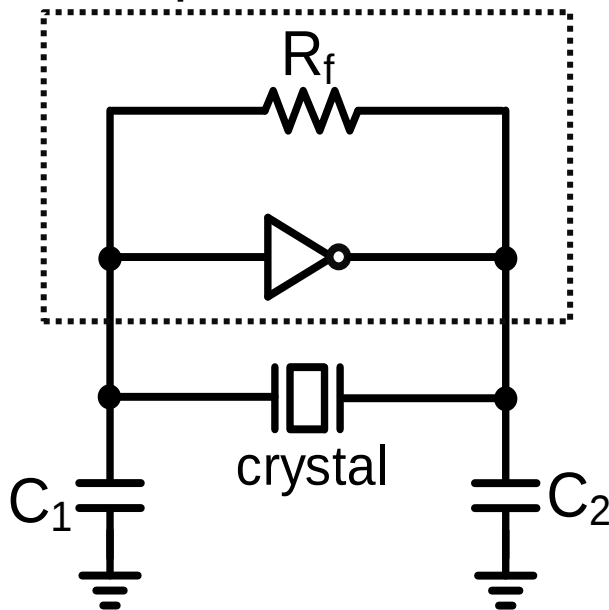
$$C_S \ll C_P, C_1, C_2$$

$$\Rightarrow \omega_o \approx \frac{1}{\sqrt{LC_S}} \approx \omega_P \approx \omega_S$$

Crystal Oscillators (Cont.)

- Pierce oscillator (one example of Colpitts oscillator)
 - ◆ Almost all digital clock oscillators are Pierce oscillators
 - ◆ Derived from Colpitts oscillator
 - Inverter with feedback resistor as an inverting amplifier

Replacement of the CE amplifier

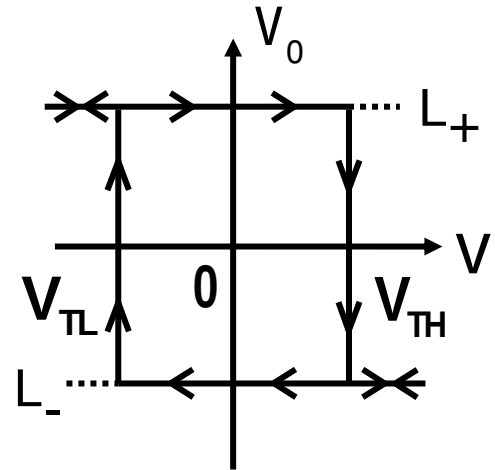
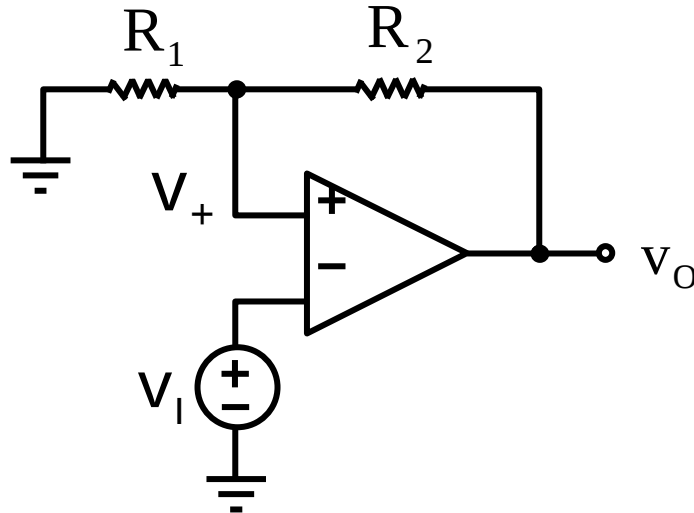


Multivibrators

- Multivibrators (3 types)
 - bistable: two stable states
 - monostable: one stable state
 - astable: no stable state
- Bistable
 - ◆ Has two stable states
 - ◆ Can be obtained by connecting an amplifier in a positive-feedback loop having a loop gain greater than unity. i.e. $\beta A > 1$ where $\beta = R_1 / (R_1 + R_2)$

Inverting Bistable Multivibrators

- ◆ Bistable circuit with clockwise hysteresis



- ◆ Clockwise hysteresis (or inverting hysteresis)

L_+ : positive saturation voltage of OPAMP

L_- : negative saturation voltage of OPAMP

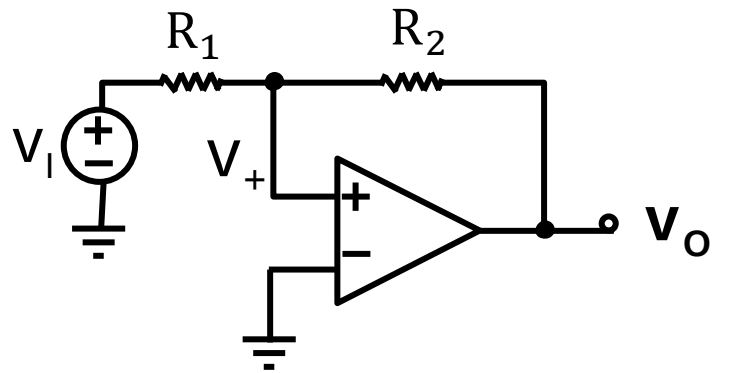
$$\Rightarrow V_{TH} = \beta L_+ = \frac{R_1}{R_1 + R_2} L_+$$

$$\Rightarrow V_{TL} = \beta L_- = \frac{R_1}{R_1 + R_2} L_-$$

→ Hysteresis width = $V_{TH} - V_{TL}$

Noninverting Bistable Multivibrators

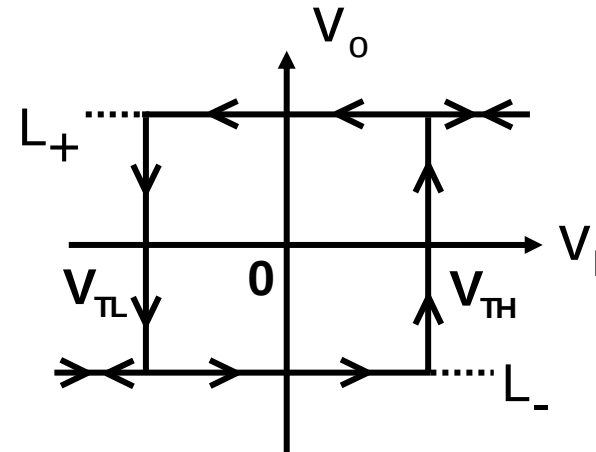
- Counter-clockwise hysteresis
- Configuration



$$V_+ = V_I \frac{R_2}{R_1 + R_2} + V_O \frac{R_1}{R_1 + R_2}$$

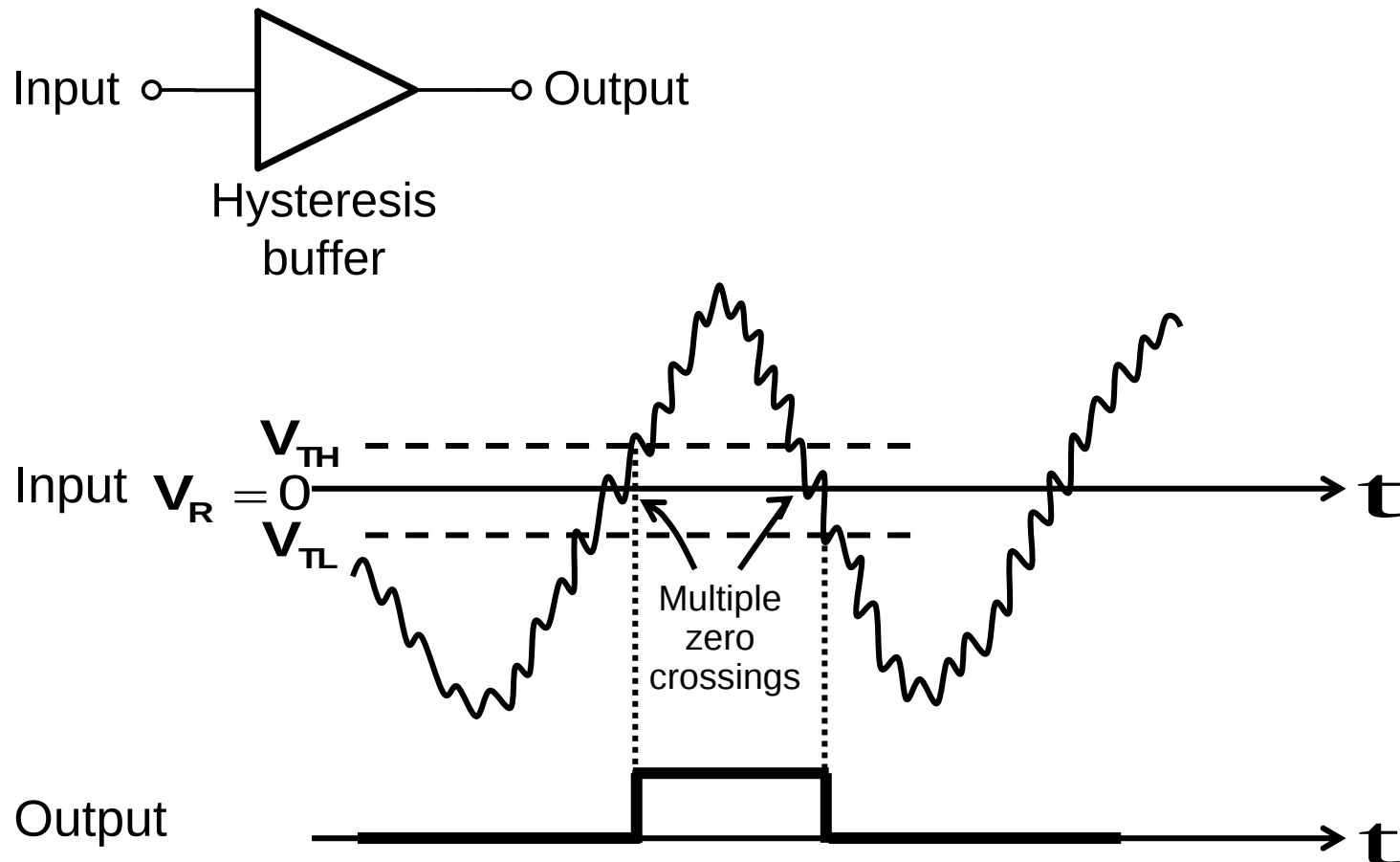
$$\text{For } v_O = L_+, v_+ = 0, v_I = V_{TL} \Rightarrow V_{TL} = -L_+ \left(\frac{R_1}{R_2} \right)$$

$$\text{For } v_O = L_-, v_+ = 0, v_I = V_{TH} \Rightarrow V_{TH} = -L_- \left(\frac{R_1}{R_2} \right)$$



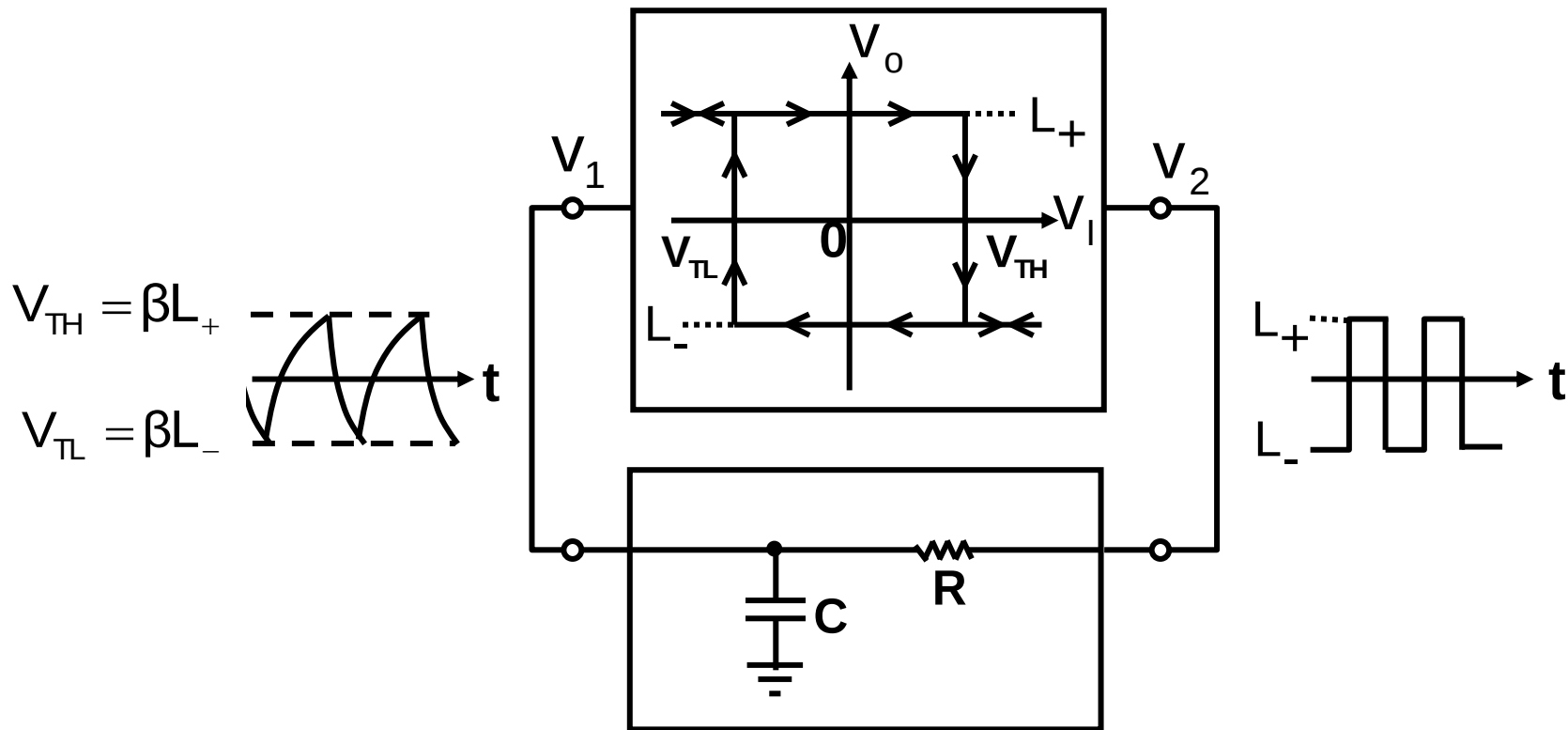
Noninverting Bistable Multivibrators (Cont.)

- Comparator characteristics with counter-clockwise hysteresis
 - ◆ Can reject interference

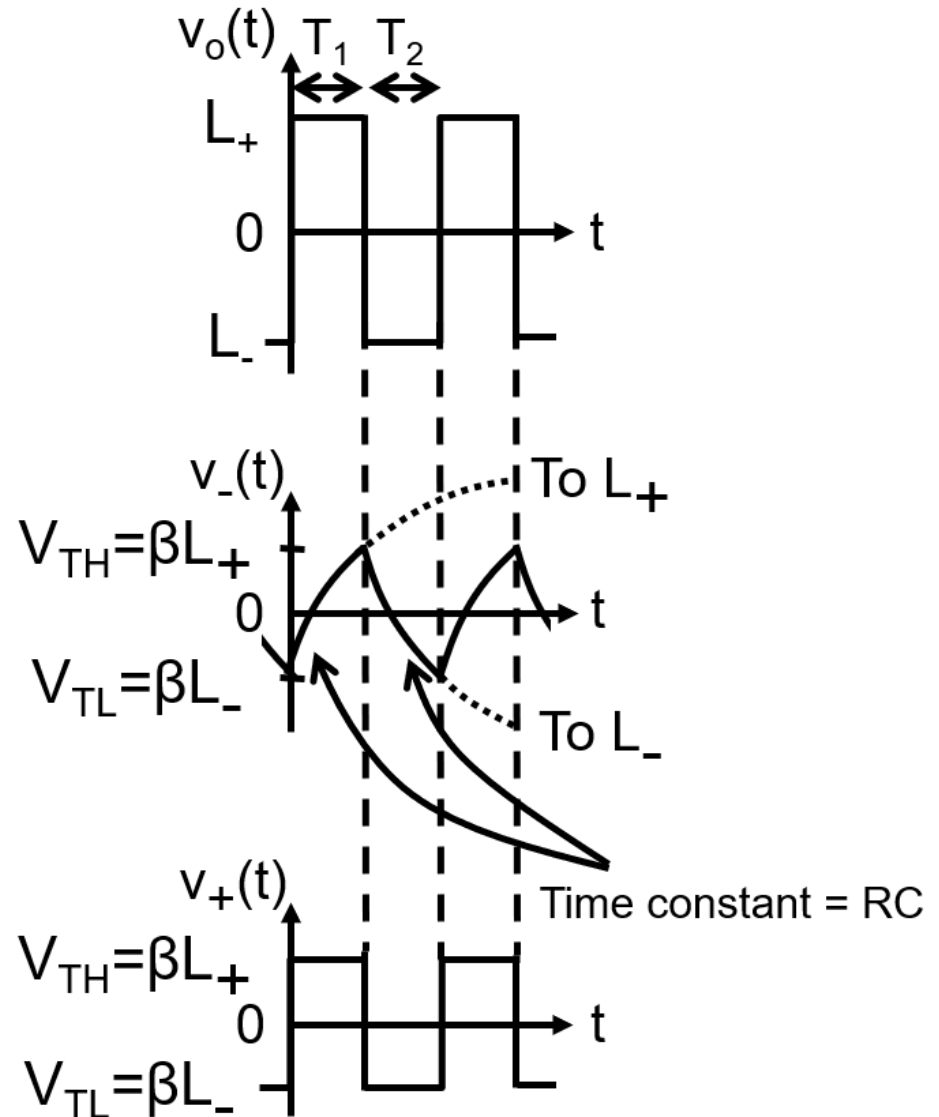
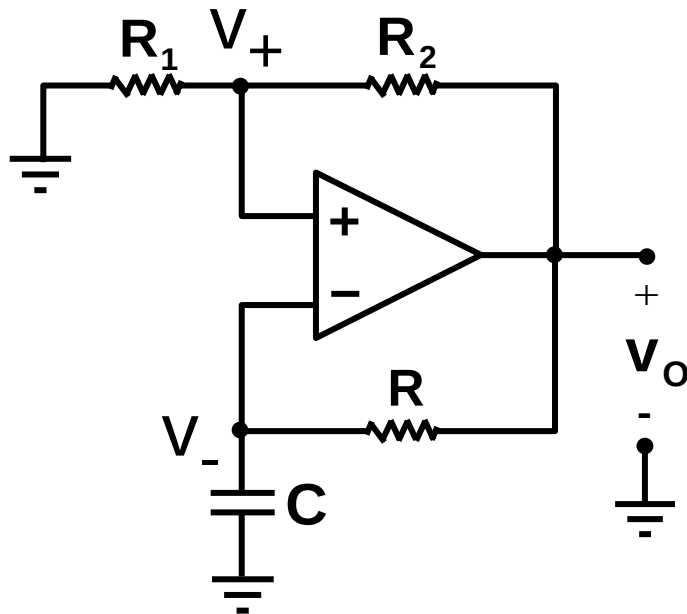


Generation of Square Waveforms Using Astable Multivibrators

- Can be done by connecting an inverting bistable multivibrator with a RC circuit in a feedback loop.



Generation of Square Waveforms Using Astable Multivibrators (Cont.)



Generation of Square Waveforms Using Astable Multivibrators (Cont.)

- During T_1

$$V_-(t) = L_+ - (L_+ - \beta L_-) e^{-t/\tau} \quad \text{where } \tau = RC, \quad \beta = \frac{R_1}{R_1 + R_2}$$

$$\text{if } V_-(T_1) = \beta L_+ \Rightarrow T_1 = \tau \ln \frac{1 - \beta(L_-/L_+)}{1 - \beta}$$

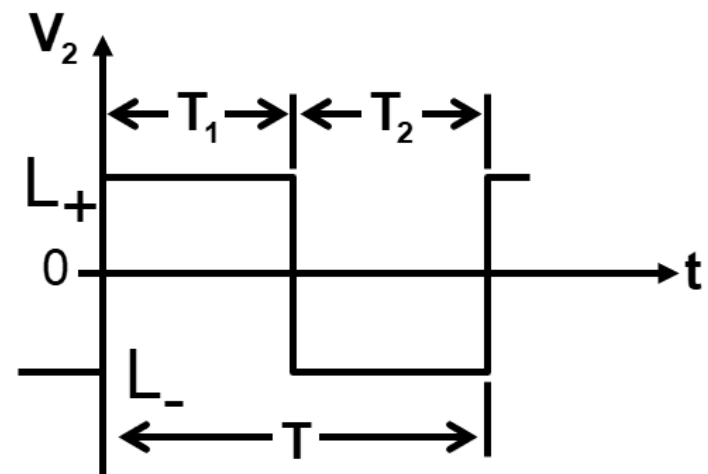
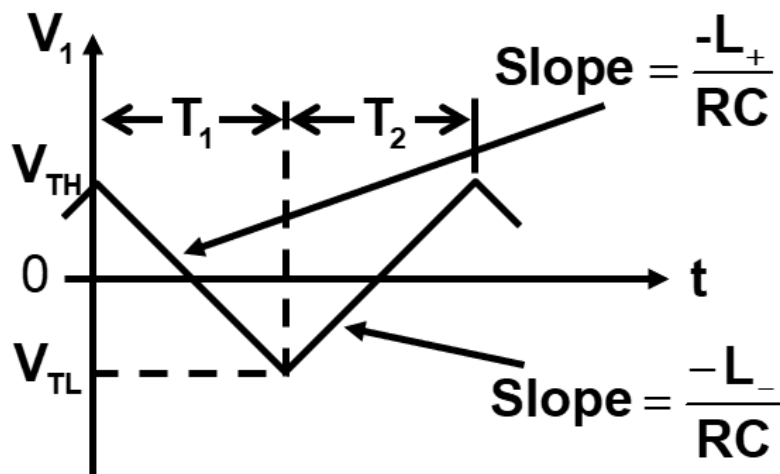
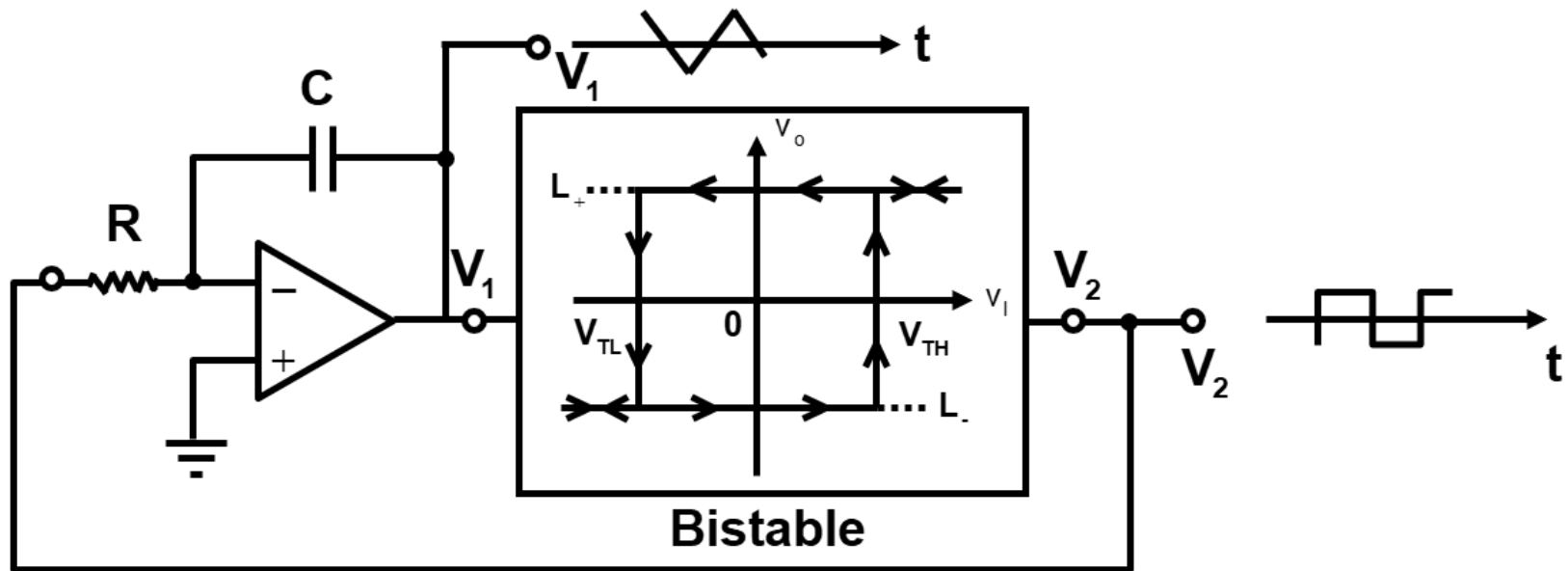
- During T_2

$$V_-(t) = L_- - (L_- - \beta L_+) e^{-(t-T_1)/\tau}$$

$$\text{if } V_-(T_2) = \beta L_- \Rightarrow T_2 = \tau \ln \frac{1 - \beta(L_+/L_-)}{1 - \beta}$$

$$T = T_1 + T_2 = 2 \tau \ln \frac{1 + \beta}{1 - \beta}; (L_+ = -L_- \text{ is assumed})$$

Generation of Triangular Waveforms Using Astable Multivibrators



Generation of Triangular Waveforms Using Astable Multivibrators (Cont.)

- During T_1

$$V_{TL} - V_{TH} = -\frac{1}{C} \int_0^{T_1} i_C dt = \frac{-L_+ T_1}{RC}; \text{ where } i_C = \frac{L_+}{R}$$

$$\Rightarrow T_1 = RC \frac{V_{TH} - V_{TL}}{L_+}$$

- During T_2

Similarly

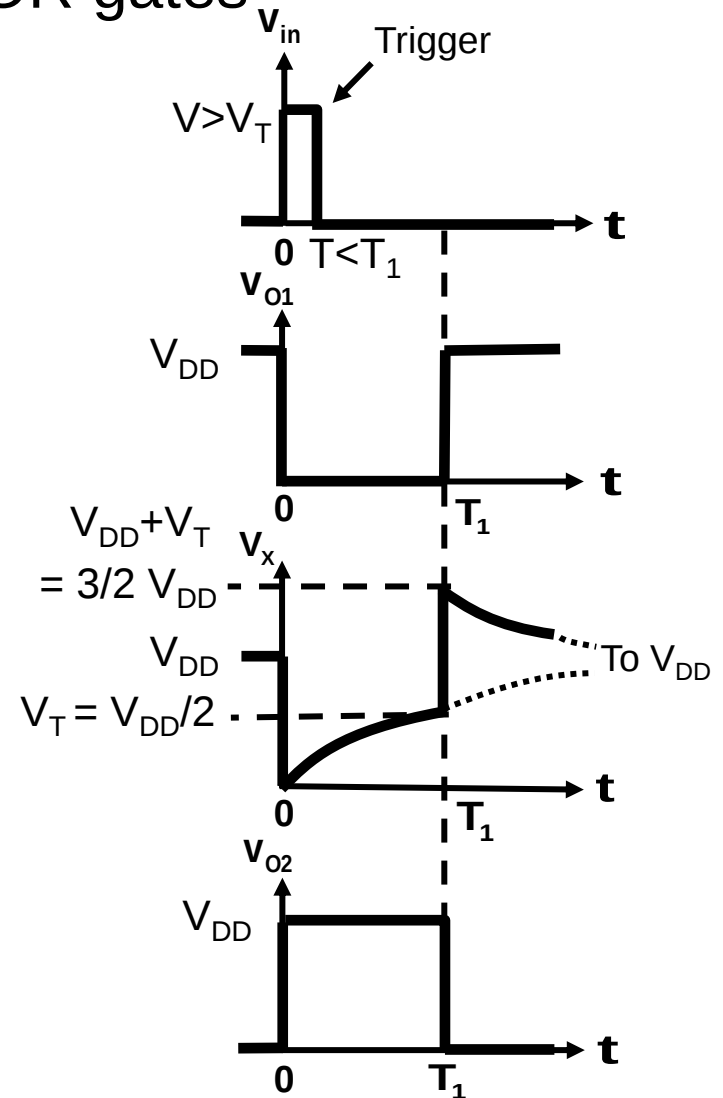
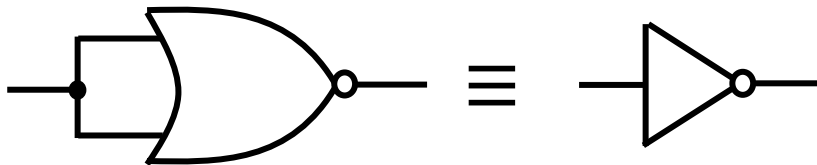
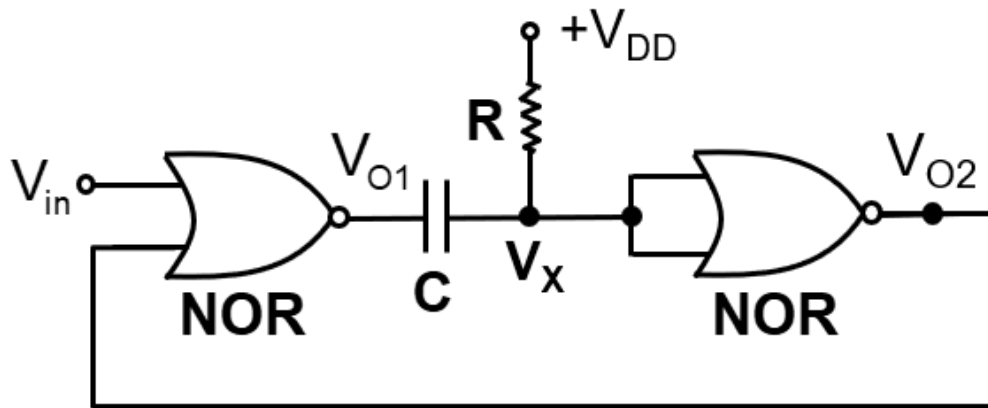
$$\Rightarrow T_2 = RC \frac{V_{TH} - V_{TL}}{-L_-}$$

- Period $T = T_1 + T_2$
- To obtain symmetrical waveforms

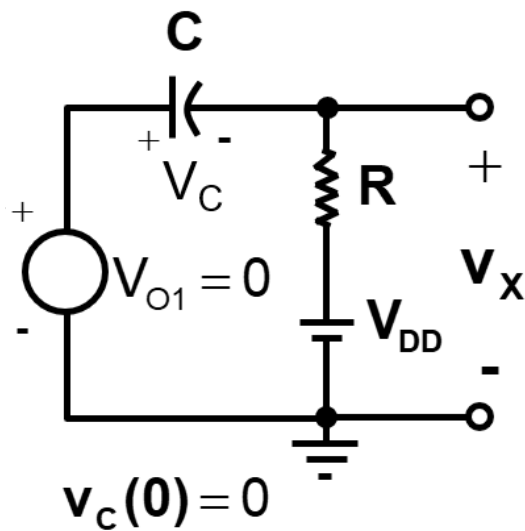
$$T_1 = T_2 \Rightarrow L_+ = L_-$$

Monostable Multivibrators

- Monostable multivibrator using NOR gates



Monostable Multivibrators (Cont.)

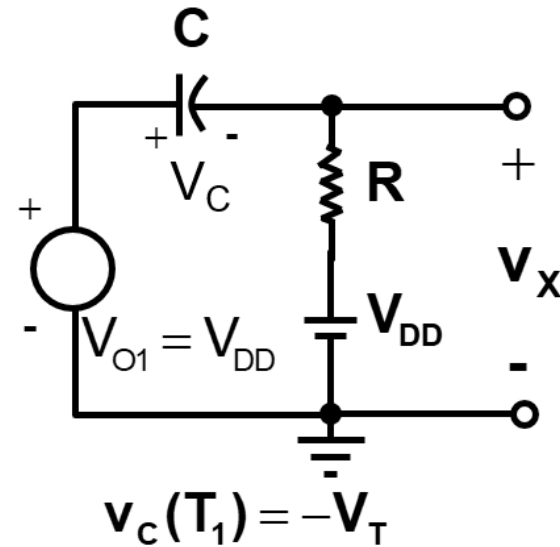


$$v_X = V_{DD} (1 - e^{-\frac{t}{RC}})$$

$$v_X(T_1) = V_T = V_{DD} (1 - e^{-\frac{T_1}{RC}})$$

$$\Rightarrow T_1 = RC \ln \frac{V_{DD}}{V_{DD} - V_T} \approx RC \ln 2 \approx 0.693RC$$

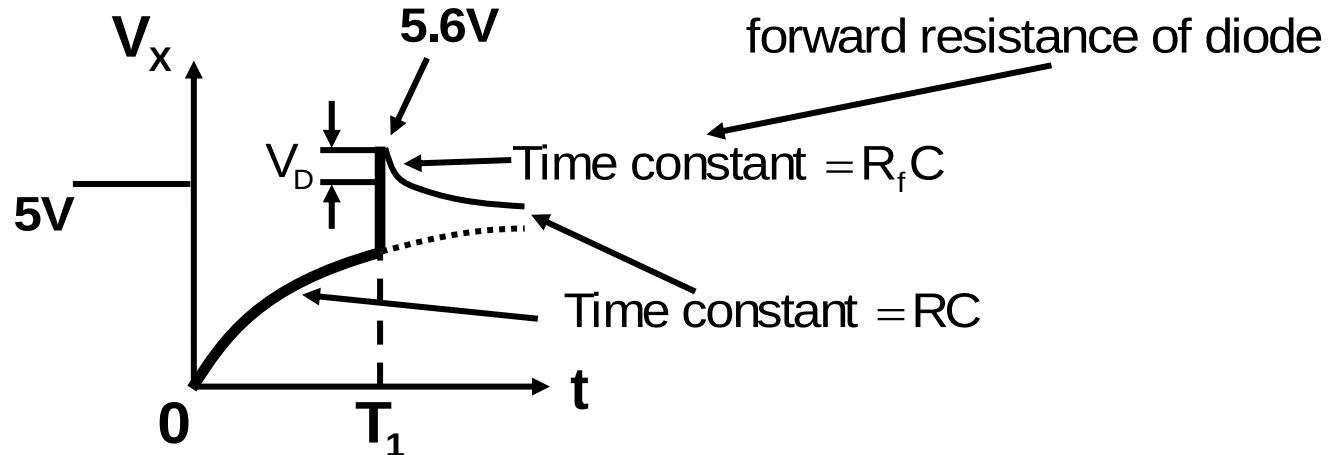
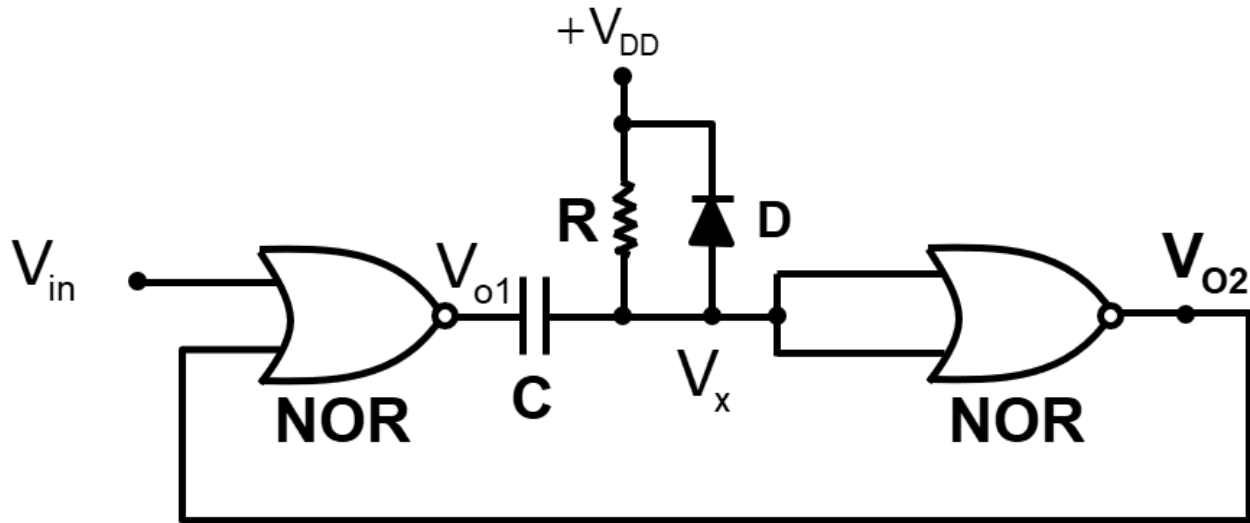
where $V_T \approx \frac{V_{DD}}{2}$; V_T is NOR gate threshold voltage



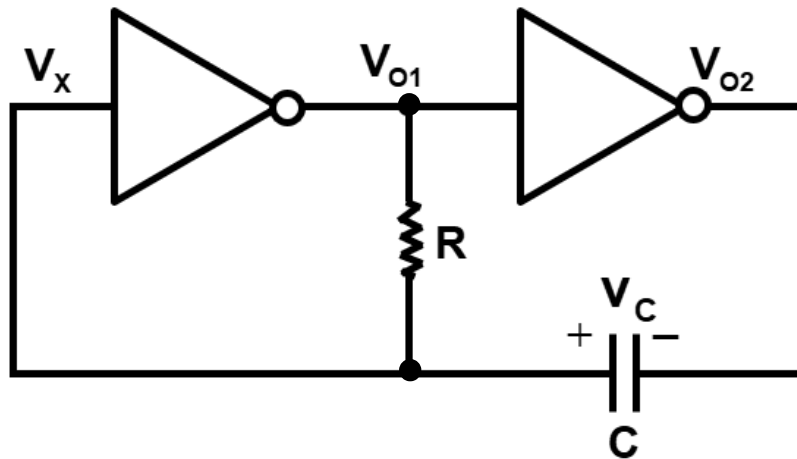
$$v_X = V_{DD} + V_T e^{-\frac{t}{RC}}$$

Monostable Multivibrators (Cont.)

- Monostable multivibrator with catching diode



Astable Multivibrator Using NOR(or Inverter) Gates



● Transient behavior

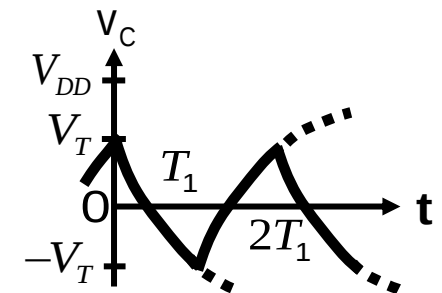
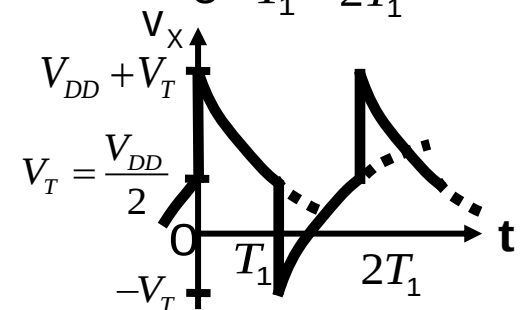
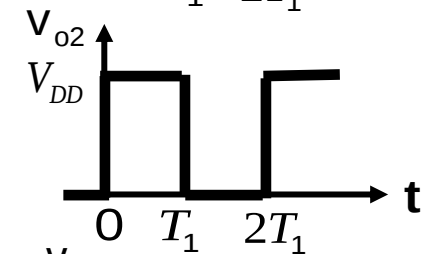
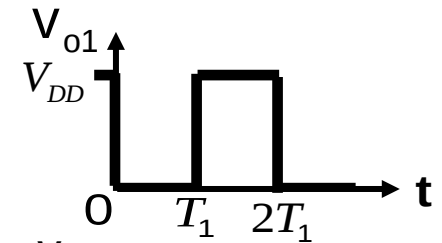
(1) $0 < t < T_1$

(i) $v_{o1}: V_{DD} \rightarrow 0$ when $t=0$

(ii) $v_{o2}: 0 \rightarrow V_{DD}$ when $t=0$

(iii) $v_x = (V_{DD} + V_T)e^{\frac{-t}{RC}}$

(iv) $v_c = v_x - V_{O2} = -V_{DD} + (V_{DD} + V_T)e^{\frac{-t}{RC}}$



Astable Multivibrator Using NOR(or Inverter) Gates (Cont.)

$$(2) T_1 < t < (T_1 + T_2)$$

$$(i) v_{o1}: 0 \rightarrow V_{DD} \quad \text{when } t = T_1$$

$$(ii) v_{o2}: V_{DD} \rightarrow 0 \quad \text{when } t = T_1$$

$$(iii) v_x = V_{DD} - (V_{DD} + V_T) e^{-\frac{(t-T_1)}{RC}}$$

$$(iv) v_c = v_x - V_{O2} = v_x = V_{DD} - (V_{DD} + V_T) e^{-\frac{(t-T_1)}{RC}}$$

Astable Multivibrator Using NOR(or Inverter) Gates (Cont.)

- Oscillation frequency

$$v_x(T_1) = V_T$$

$$\Rightarrow (V_{DD} + V_T)e^{-\frac{T_1}{RC}} = V_T$$

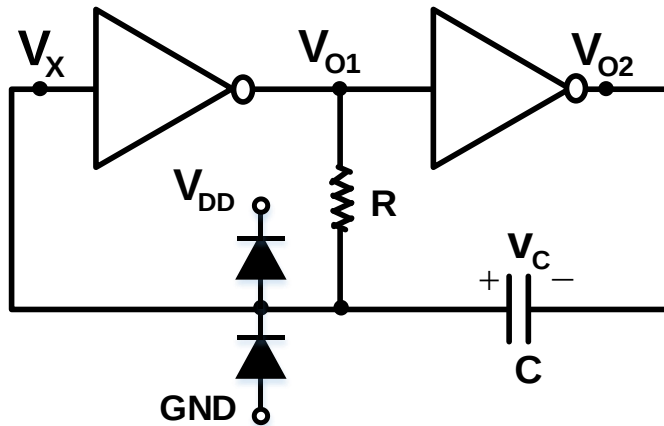
$$\Rightarrow T_1 = RC \ln \frac{V_{DD} + V_T}{V_T}$$

$$\text{If } V_T = \frac{V_{DD}}{2}, \text{ then } T_1 = RC \ln 3 \text{ and } T_2 = RC \ln 3$$

$$\text{oscillation frequency } f_0 = \frac{1}{2RC \ln 3} \approx \frac{0.455}{RC}$$

Astable Multivibrator Using NOR(or Inverter) Gates (Cont.)

- ◆ With catching diode, which can be on-chip and also needed for ESD protection



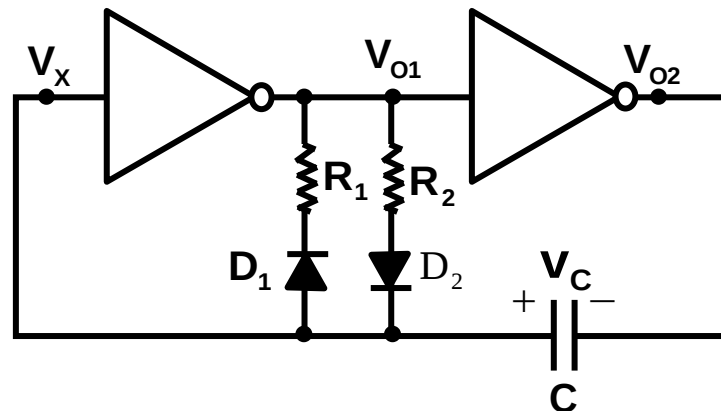
$$T_1 = T_2 = RC \ln 2$$

$$f_0 = \frac{1}{2RC \ln 2} \approx \frac{0.721}{RC}$$

- ◆ Asymmetrical square wave

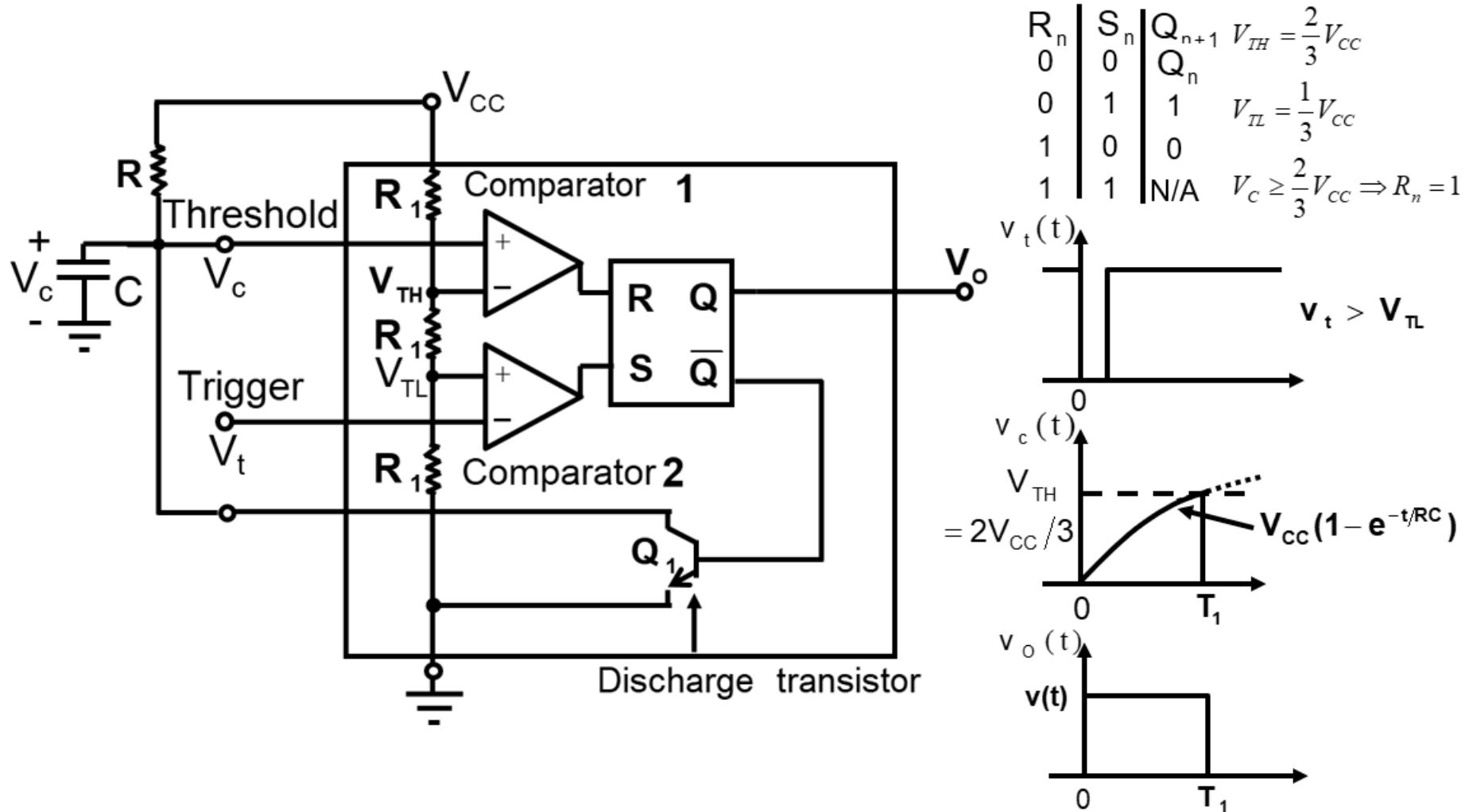
$$(i) V_T \neq \frac{V_{DD}}{2}$$

$$(ii) R_1 \neq R_2$$



Reading Assignment: The 555 IC Timer

- Used as a monostable multivibrator



Reading Assignment: The 555 IC Timer (Cont.)

◆ For $0 \leq t \leq T_1$

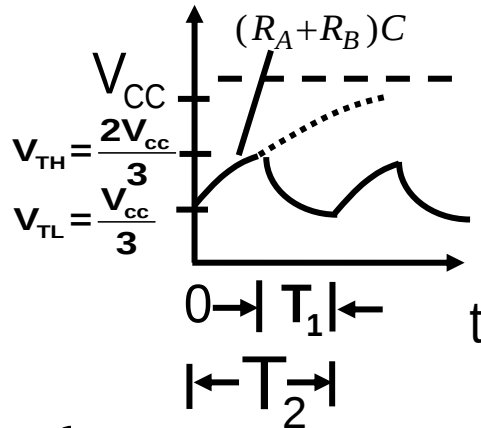
$$v_C(t) = V_{CC} - [V_{CC} - V_C(0)]e^{-\frac{t}{RC}} \quad (V_C(0) \approx V_{CE(sat)} \approx 0)$$

◆ For $t = T_1$, $v_C(T_1) = V_{TH} = \frac{2V_{CC}}{3}$

$$\Rightarrow T_1 = RC \ln \frac{V_{CC} - V_C(0)}{\frac{V_{CC}}{3}} \approx RC \ln 3 \quad (V_C(0) \approx 0)$$

Reading Assignment: The 555 IC Timer (Cont.)

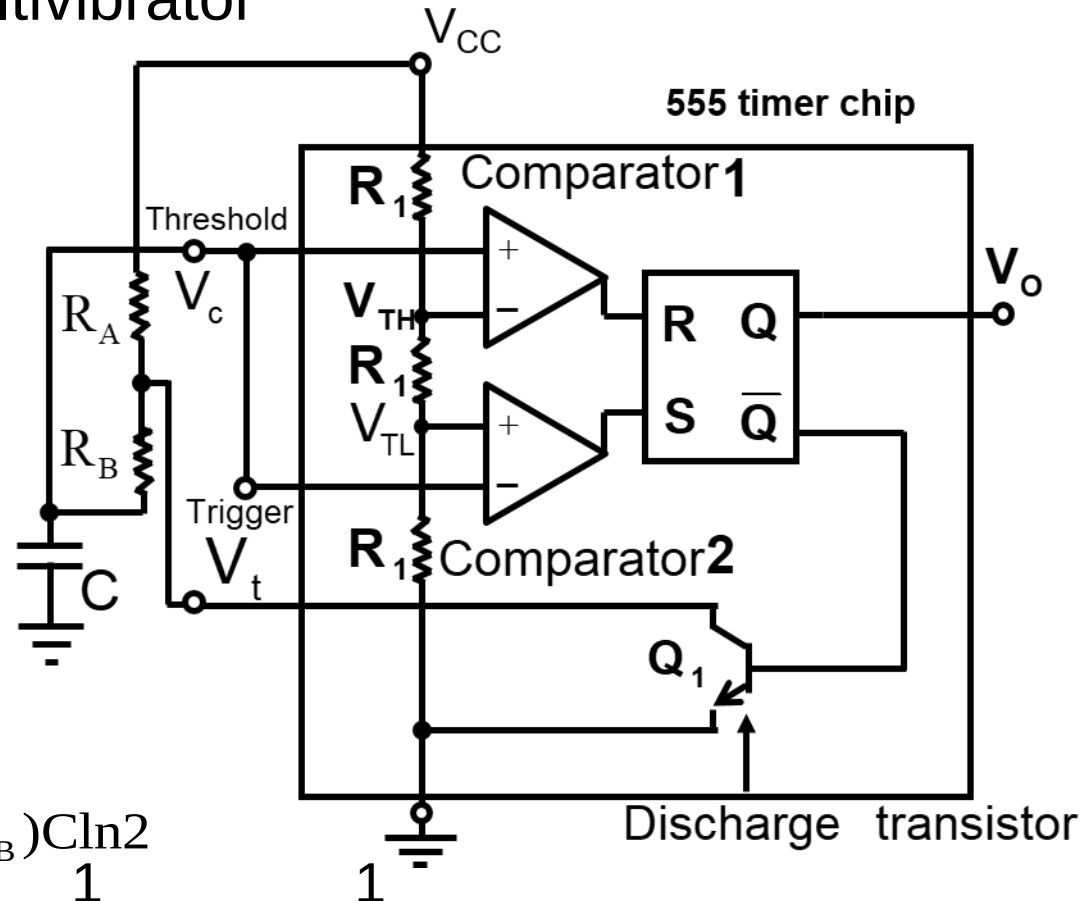
- Used as an astable multivibrator



$$\begin{cases} V_c \geq \frac{2V_{cc}}{3} \Rightarrow S = 0, R = 1 \\ V_c \leq \frac{V_{cc}}{3} \Rightarrow S = 1, R = 0 \\ \frac{V_{cc}}{3} \leq V_c \leq \frac{2V_{cc}}{3} \Rightarrow S = R = 0 \end{cases}$$

$$T_1 = R_B C \ln 2, T_2 - T_1 = (R_A + R_B) C \ln 2$$

- Oscillation frequency $f = \frac{1}{T_2} = \frac{1}{(R_A + 2R_B) C \ln 2}$



Appendix

- Nonlinear amplitude control
- Sine-wave shaper
- Precision rectifier circuits
- Precision full-wave rectifiers
- Peak rectifier
- Crystal oscillators

Nonlinear Amplitude Control

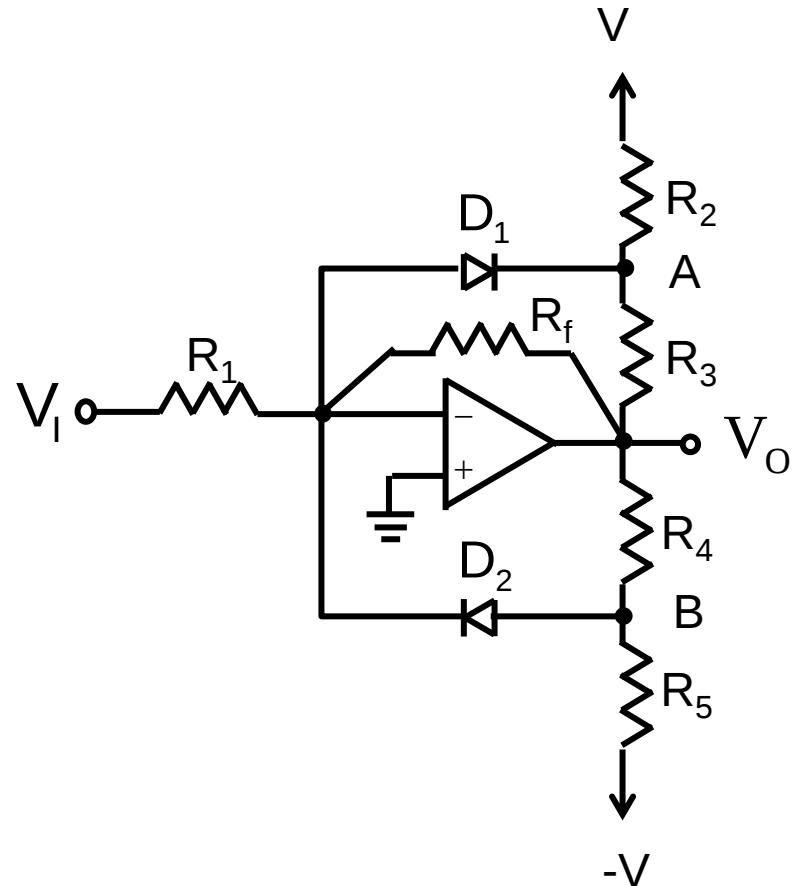
- Limiter circuit for amplitude control

- ◆ Linear region

$$V_O = -\left(\frac{R_f}{R_1}\right)V_i$$

$$V_A = V \frac{R_3}{R_2 + R_3} + V_O \frac{R_2}{R_2 + R_3}$$

$$V_B = -V \frac{R_4}{R_4 + R_5} + V_O \frac{R_5}{R_4 + R_5}$$



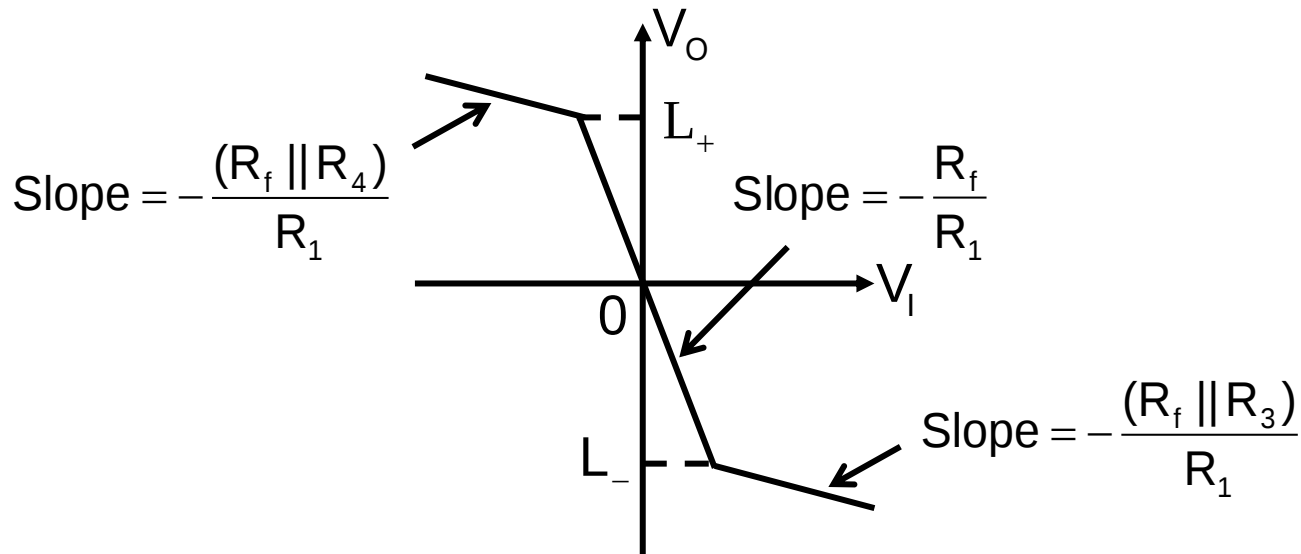
Nonlinear Amplitude Control (Cont.)

◆ Nonlinear region

$$V_O|_{V_A=-V_D}=L_- \Rightarrow \text{for } V_A=-V_D; \frac{R_2}{R_2+R_3}L_- + \frac{R_3}{R_2+R_3}V = V_A = -V_D$$

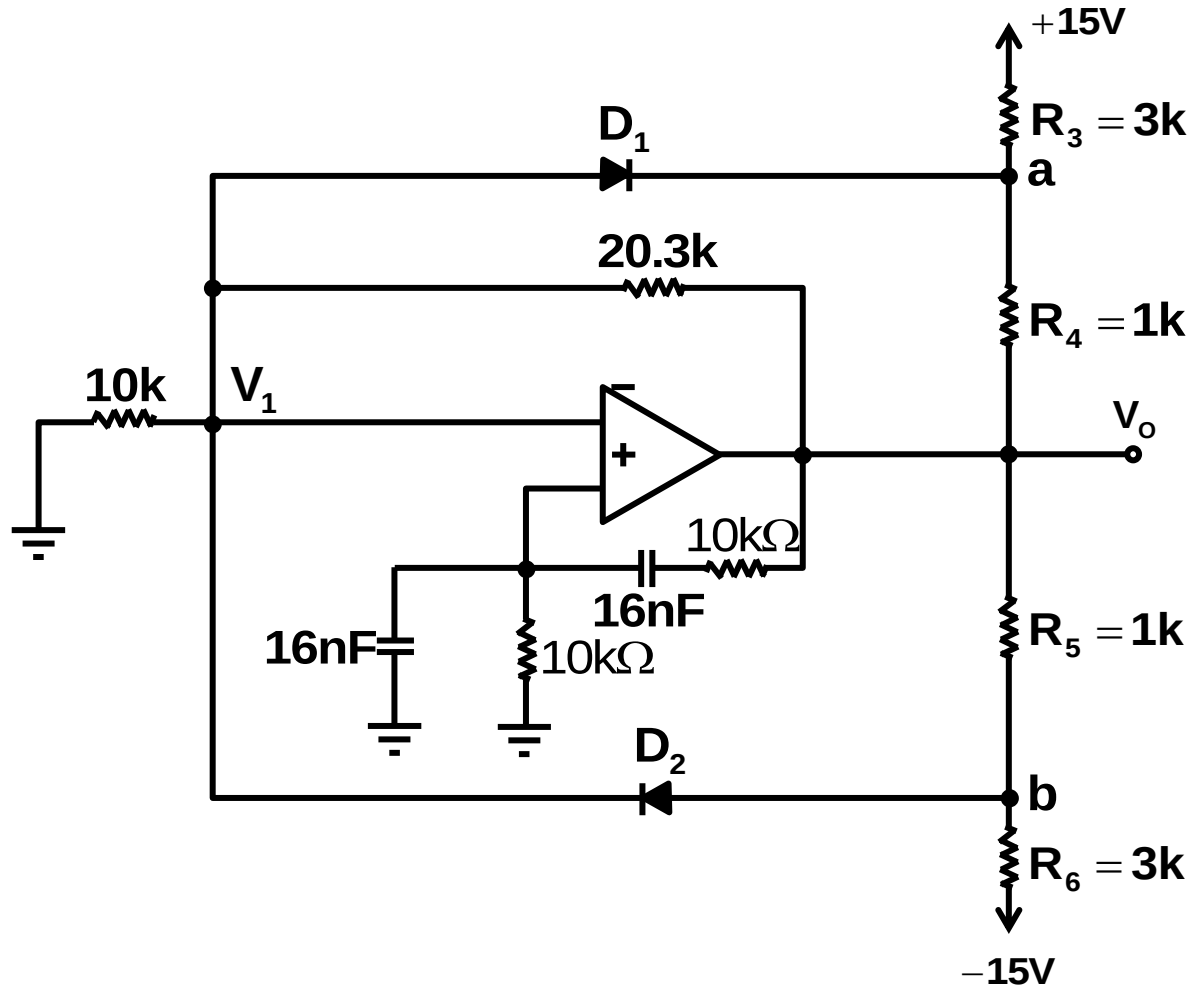
$$\Rightarrow L_- = \frac{R_2+R_3}{R_2}(-V \frac{R_3}{R_2+R_3} - V_D) = -V \frac{R_3}{R_2} - V_D(1 + \frac{R_3}{R_2});$$

$$\text{similarly, } V_O|_{V_B=V_D} = L_+ = V \frac{R_4}{R_5} + V_D(1 + \frac{R_4}{R_5})$$



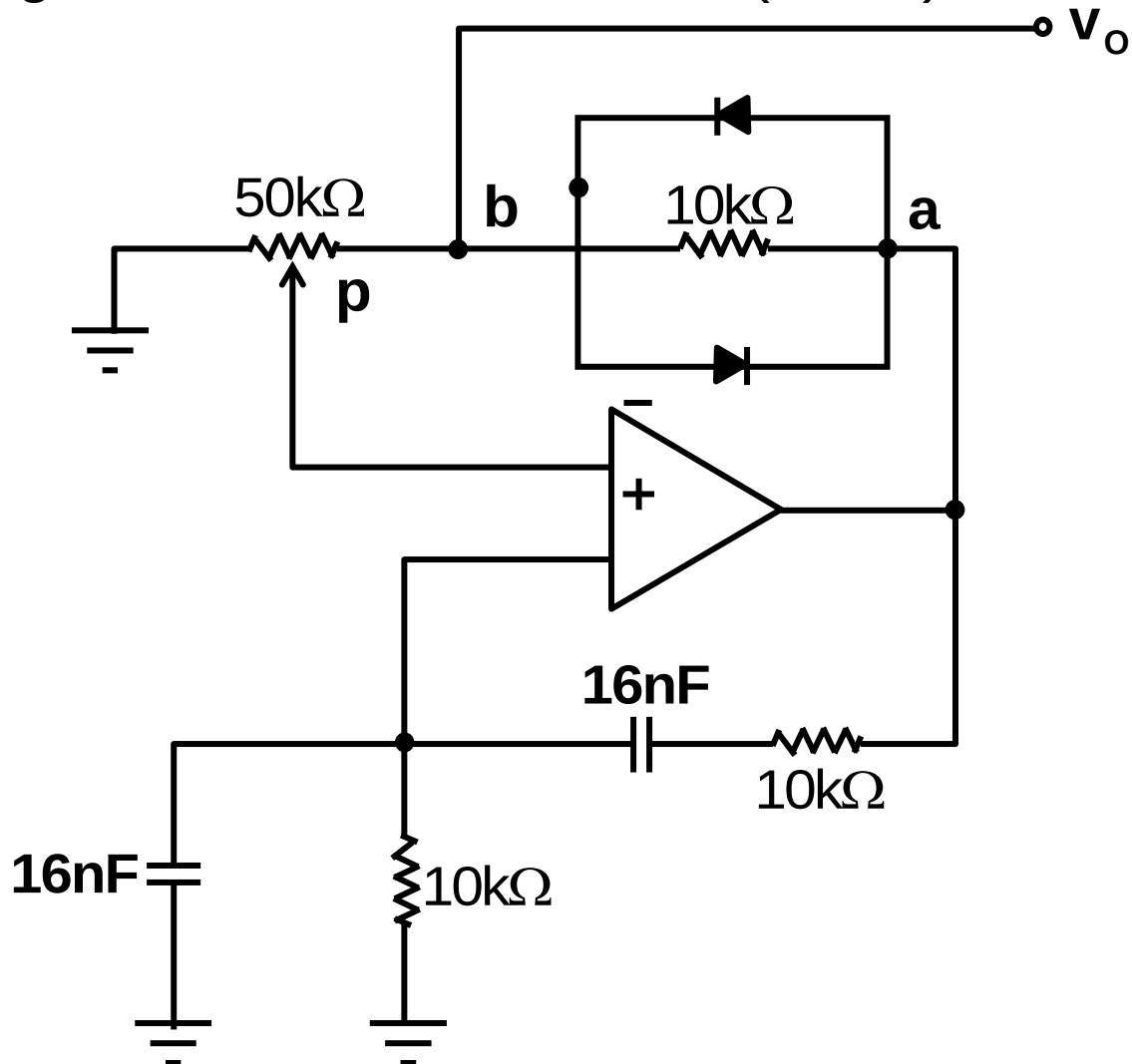
Nonlinear Amplitude Control (Cont.)

- Wien-bridge oscillator with a limiter



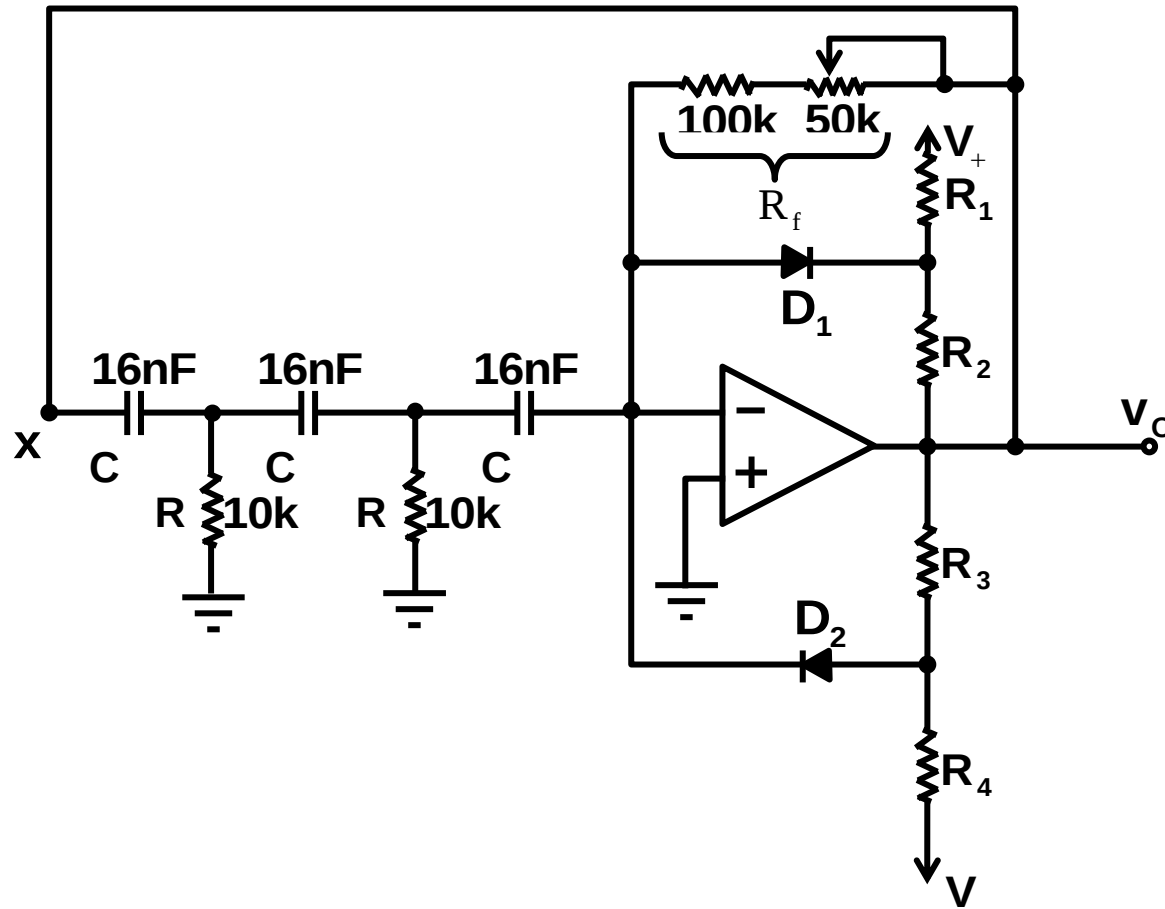
Nonlinear Amplitude Control (Cont.)

- Wien-bridge oscillator with a limiter (Cont.)



Nonlinear Amplitude Control (Cont.)

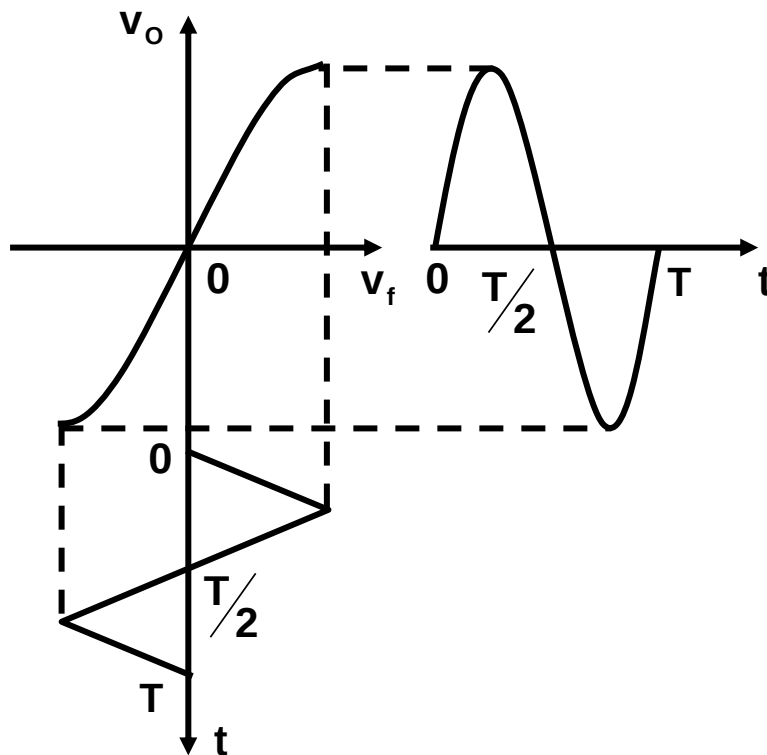
- Phase-shift oscillator



Sine-Wave Shaper

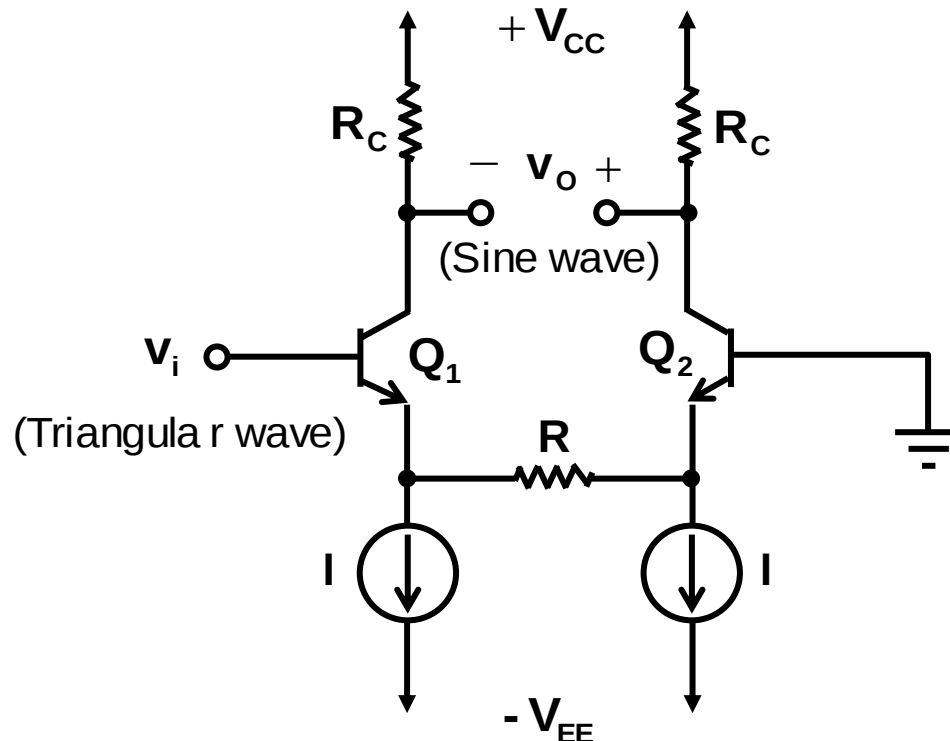
- Shape a triangular waveform into a sinusoid
- Extensively used in function generators
- Note: linear oscillators are

not cost-effective for low frequency application
not easy to time over wide frequency ranges



Sine-Wave Shaper (Cont.)

- Nonlinear-amplification method
 - ◆ For various input values, their corresponding output values can be calculated
 - Transfer curve can be obtained and is similar to



Sine-Wave Shaper (Cont.)

- Breakpoint method
 - ◆ Piecewise linear transfer curve
 - ◆ Low-valued R is assumed $\rightarrow V_1$ and V_2 are constant

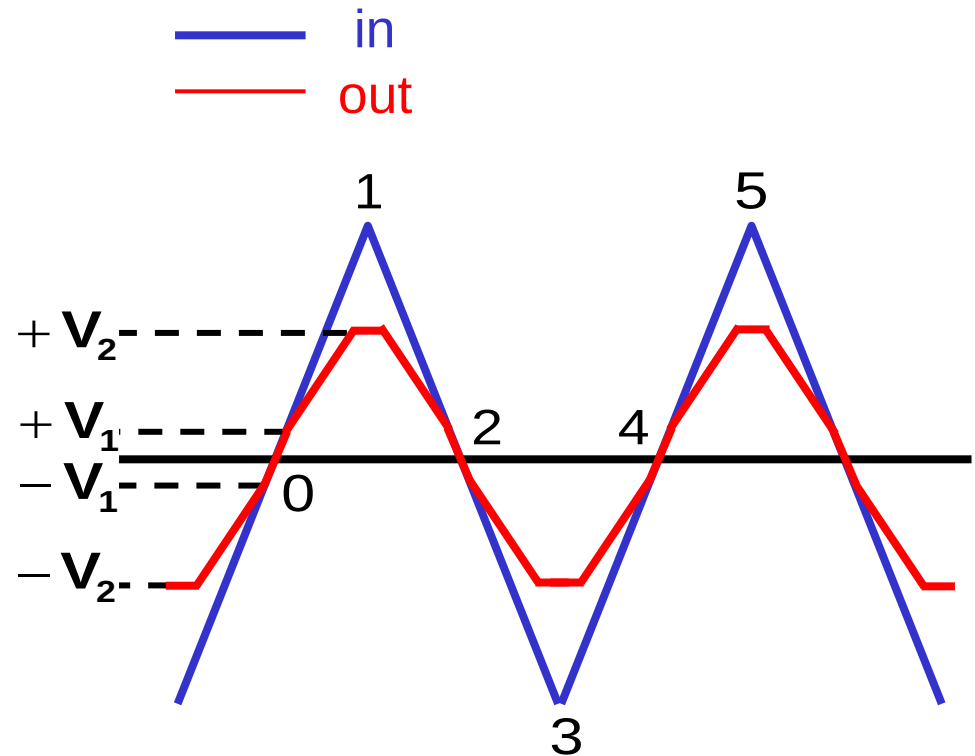
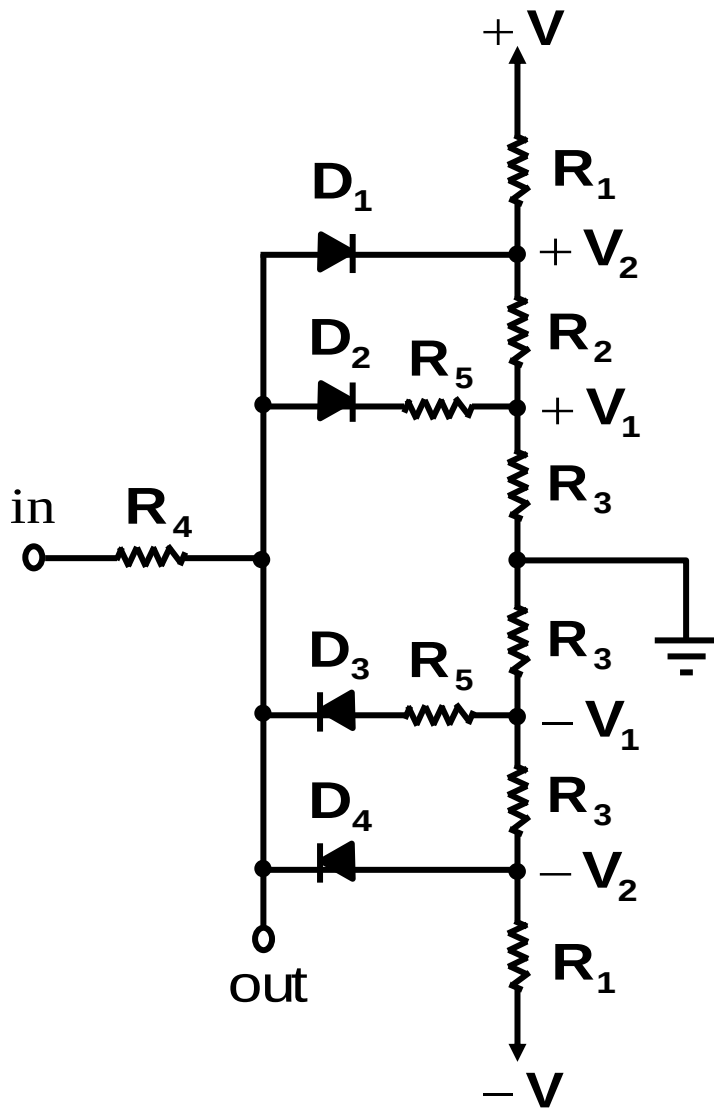
$$V_1 < V_{in} < V_1 \Rightarrow V_{out} = V_{in}$$

$$V_1 < V_{in} < V_2 \Rightarrow D_2 \text{ is on (voltage drop } V_D)$$

$$\Rightarrow V_0 = V_1 + V_D + (V_{in} - V_D - V_1) \frac{R_5}{R_5 + R_4}$$

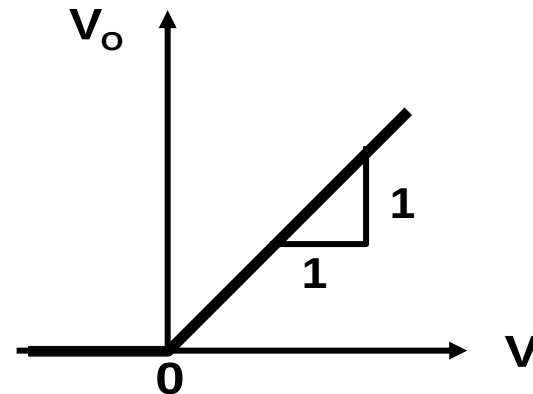
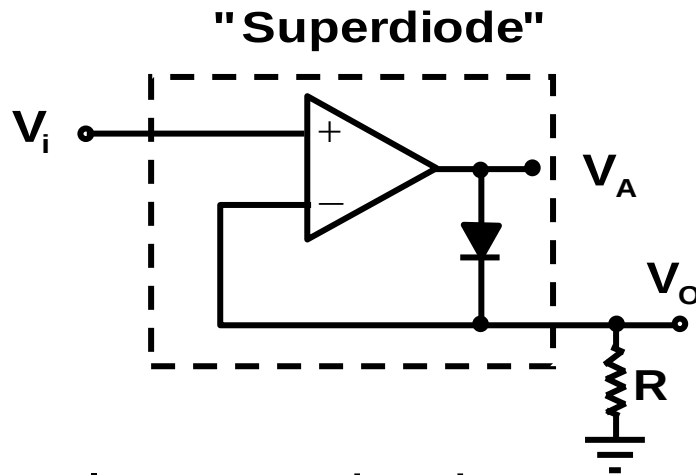
$$V_2 < V_{in} \Rightarrow D_1 \text{ is on} \Rightarrow \text{limit } V_0 \text{ to } V_2 + V_D$$

Sine-Wave Shaper (Cont.)

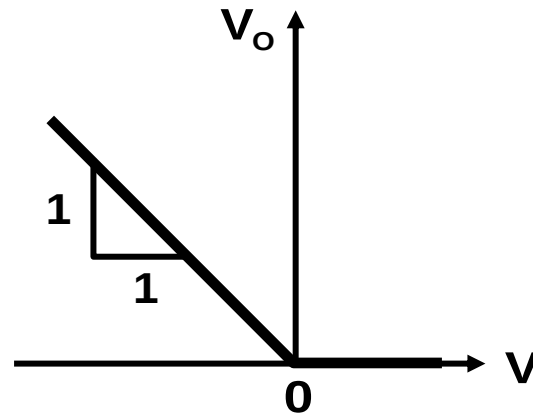
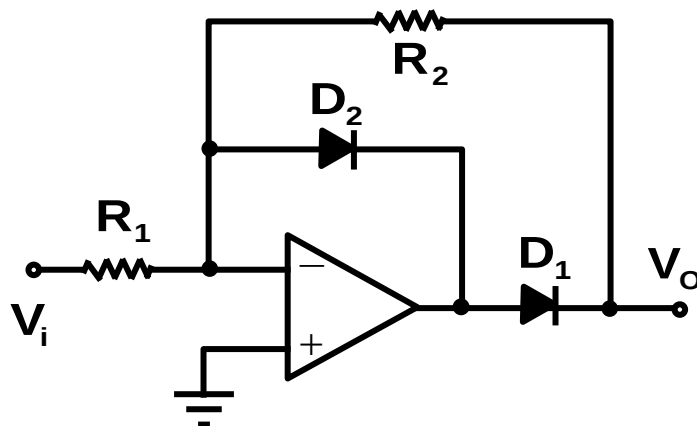


Precision Rectifier Circuits

- Precision half-wave rectifier --- “superdiode”

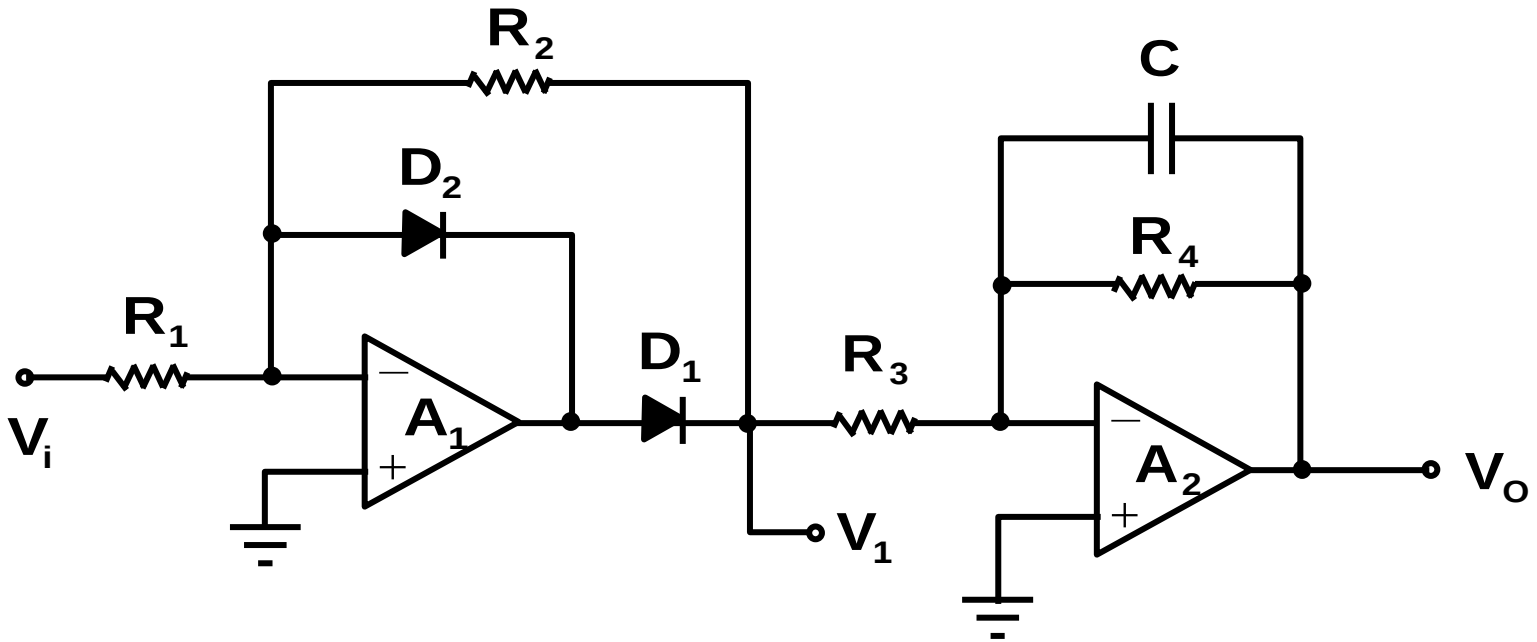


- An alternate circuit



Precision Rectifier Circuits (Cont.)

- ◆ Application : Measure AC voltages



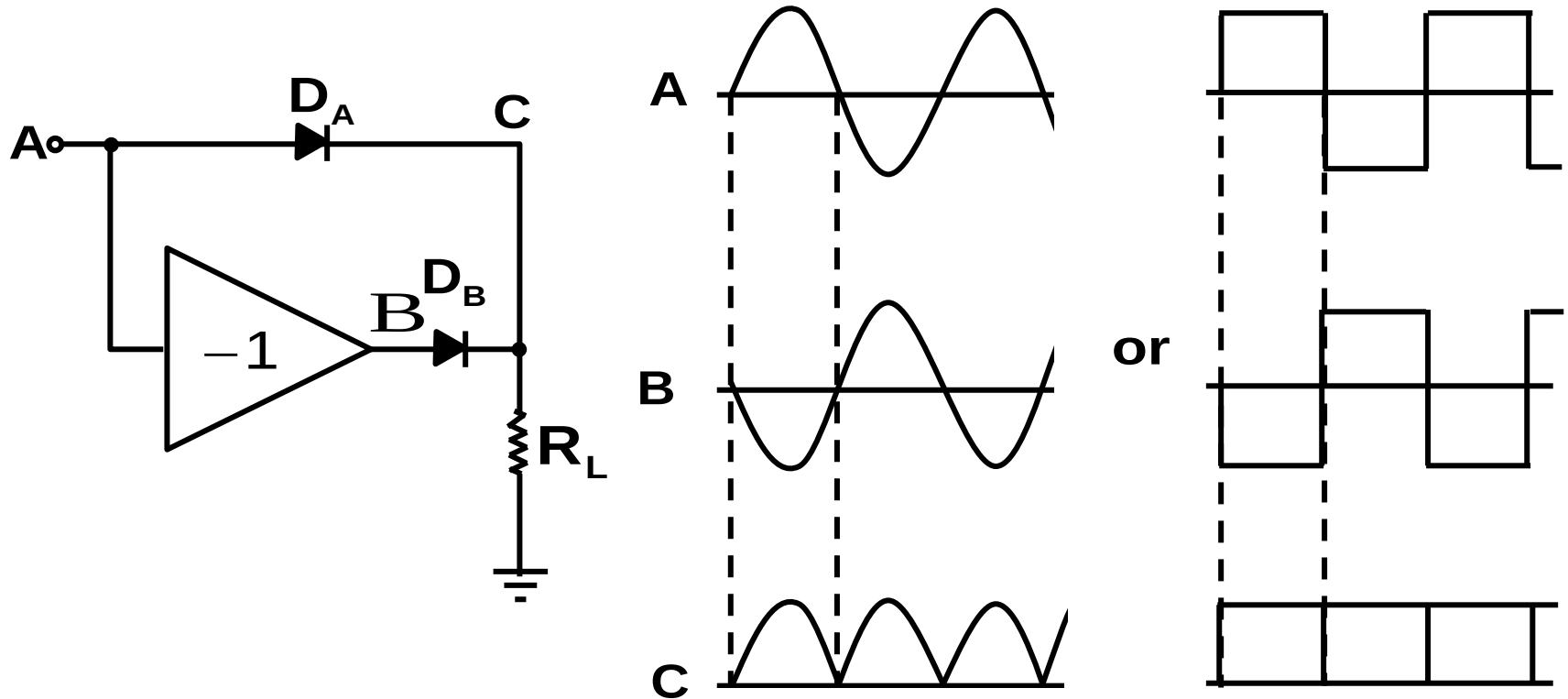
Average $V_1 = \frac{V_P}{\pi} \frac{R_2}{R_1}$; where V_p is the peak amplitude of an input sinusoid

Precision Rectifier Circuits (Cont.)

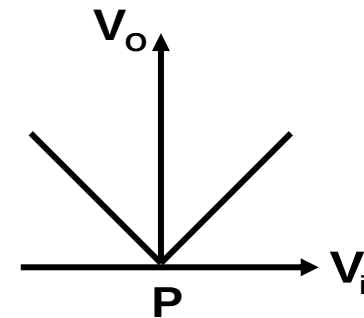
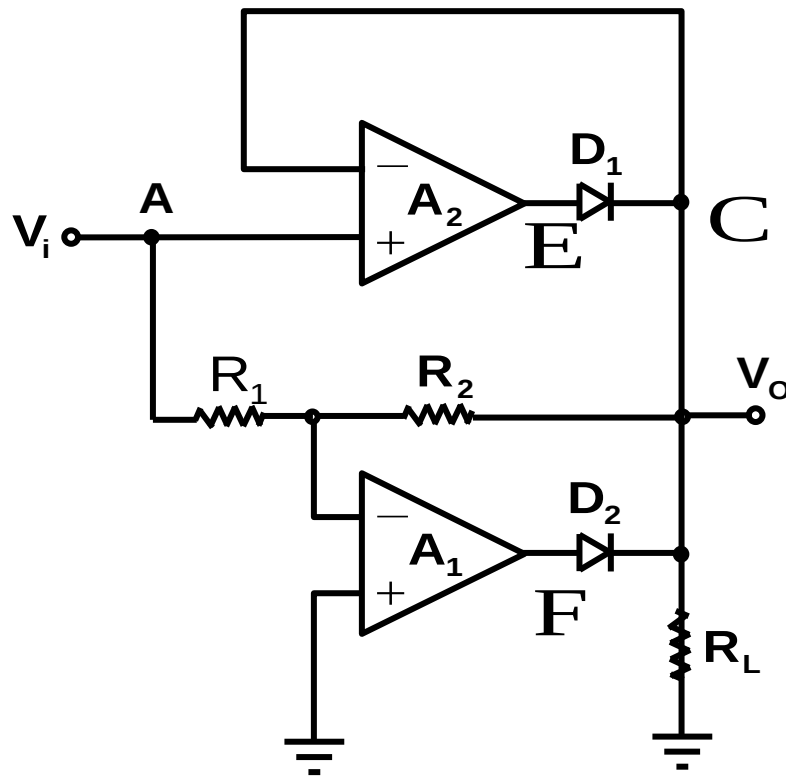
If $\frac{1}{R_4 C} \ll \omega_{\min}$; $\omega_{\min}$ is the lowest expected frequency of the input sine wave

$$\Rightarrow V_o = -\frac{V_P}{\pi} \frac{R_2}{R_1} \frac{R_4}{R_3}$$

Precision Full-Wave Rectifiers

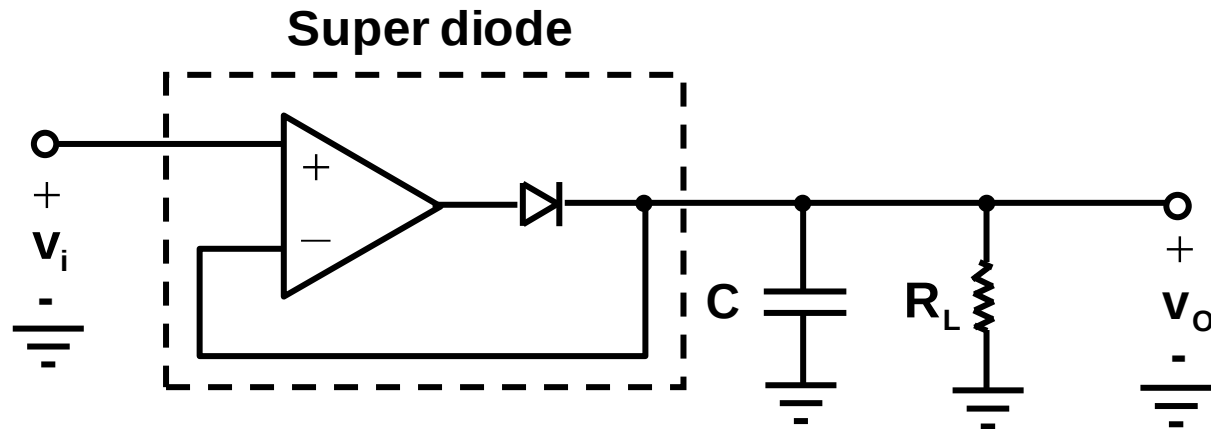


Precision Full-Wave Rectifiers (Cont.)



Peak Rectifier

- With load



- Buffered

